Field of Pipe Dreams: Minimizing Maintenance Cost for Hand-Moved Irrigation Systems

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Contents

The "hand-move" irrigation system is widely used on small fields due to its versatility and mobility. The primary defect of the system is the large maintenance time it requires. When designing such a system, therefore, the primary objective is to minimize the required maintenance time without sacrificing water distribution uniformity. In this particular problem, we are asked to minimize the maintenance time of a hand-move irrigation system under the following constraints:

- No part of the field should receive more than 0.75 cm of water,
- Each part of the field should receive at least 2 cm of water every 4 days, and
- The water should be applied 'as uniformly as possible'.

There are two ambiguities in these requirements; namely, the definition of maintenance time and the vague distribution uniformity requirement. We define 'maintenance time' provisionally to be the number of 20-meter pipes that must be moved per unit time. We will discuss balancing water uniformity and ease of maintenance later.

We took the following steps in solving this problem:

- We first calculated the pressure loss in the pipes due to friction, and concluded that the small flow (and hence small velocity in the pipes) renders this effect insignificant.
- We calculated the velocity of water ejected from a single sprinkler, used this to calculate the flow in the sprinkler.
- We next modeled a sprinkler head as a large collection of water droplets ejected from a common point, with varying velocities and radii. This information was used to determine the radial distribution of water ejected from a sprinkler head.
- Finally, we used the radial distributions we calculated as input in an algorithm designed to find the most efficient method of setting up and scheduling the irrigation system.

We begin by developing a physical model of the components of our irrigation system. To do this, we must model the following:

- The pressure loss along the pipe set, and
- the radial distribution of water ejected by the sprinkler.

With this information we can use an algorithm described later to determine the minimum maintenance time configuration.

1 Field and pipe set layout

The length of our pipe set is the first parameter we must choose. The primary lengths available are 20, 40, and 80 meters (noting that the length must be an integral multiple of 20). Each length comes with its own advantages and disadvantages; for example, the 20 meter long pipe is easy to move, but requires a lot of movements. We assume that the pipes can be detached and moved as individual segments, and that the detachment time is insignificant compared to the overall time required to move the pipes. Then the total time required to move the pipe set is proportional to the number of pipes; i.e., $T_m(l) = Cl$. Because the number of times we need to move the pipe set is inversely proportional to the number of pipes ($N_m(l) = Kl^{-1}$), the total move time should be approximately the same for each pipe set length:

$$\text{total move time} = (N_m)(T_m) = (Kl^{-1})(Cl) = CK \tag{1}$$

Thus the number of pipe moves per unit time is equal for each pipe set length. However, if one uses the 20-meter or 40-meter length pipe set, one must carry out these pipe moves in smaller bunches than with the 80-meter pipe set, causes additional time to be lost in returning to the field each time the pipes must be moved. Thus our goal of minimizing the total required maintenance time forces us to use the 80-meter pipe set. Then to supply water efficiently to our pipe set, we place the water source along a narrow edge of the field.

2 Pressure loss

2.1 Friction losses

The first pressure loss term we consider is the friction loss. The quality of fluid flow is given by the Reynolds number:

$$\mathbf{R} = \frac{ulp}{\mu} \tag{2}$$

Where u is a characteristic fluid velocity, l is a characteristic length, and μ is the viscosity of the fluid. Here our fluid flow ranges from 20 L/min to 150 L/min, as we will see later. We choose as a characteristic fluid flow 100 L/min, which gives a characteristic velocity of

$$u = \frac{\text{Flow}}{\text{Area}} = \frac{.10/60 \text{ m}^3/\text{s}}{\pi(.05)^2 \text{ m}^2} = 0.212 \text{ m/s} \tag{3}$$

The characteristic length is given by the diameter of the pipe, 0.1 m. Thus

$$\mathbf{R} = \frac{(.212 \text{ m/s})(.1 \text{ m})(1000 \text{ kg/m}^3)}{(.00089 \text{ kg/m s})} = 23820 \gg 2000 \tag{4}$$

Thus we may assume that our fluid flow is always turbulent.

The head loss due to friction is approximated well [1] by the Darcy-Weisbach equation

$$h_f/L = \frac{f}{D}\frac{v^2}{2g} \tag{5}$$

where h_f/L is the head loss per unit length, f is the friction factor, D is the pipe diameter, and v is the fluid velocity. The friction factor depends on the Reynolds number and the absolute roughness, which for aluminum [2] is 1.5 microns. Using the Moody chart on p. 293 of [1], we see that the f is approximately 0.015. Thus

$$h_f/L = \frac{0.015}{0.1}\frac{(.212)^2}{2(9.81)} = 3.44 * 10^{-4} \tag{6}$$

The maximum possible length of pipe we will consider is 80 m, and thus the maximum head loss due to friction is 0.0275 m. This head loss manifests itself entirely as a loss in pressure along the pipe by the equation of continuity. Thus our head loss results in a pressure drop of

$$\Delta p = h_f \rho g = (0.0275)(1000)(9.81) = 270 \text{ Pa.} \tag{7}$$

We see that frictional losses are not important, and may be ignored.

2.2 Minor losses

The other pressure loss term we must calculate is the pressure drop due to minor losses. These arise from the intersections of the sprinkler heads with the main tube. We will consider this intersection as a tee-junction, and calculate the head loss for the in-line flow of the tee-junction. According to [1], this is given by

$$h_t = 0.5\frac{v_f^2}{2g} = \frac{1}{4}\frac{(.212)^2}{9.81} = 0.0011 \text{ m} \tag{8}$$

Because the initial head is $P_i/\rho g = 42.81$ m, we see that the minor losses are also insignificant and may be ignored.

We conclude that there is very little pressure loss across the main pipe, and that we may take the pressure to be uniform throughout the system.

3 Velocity and volume of water ejected from nozzle

The previous calculation showed that the pressure is essentially uniform throughout the pipe system. Thus we conclude that the volume and velocity of water ejected from each sprinkler is the same as from the others. We must next determine the ejection velocity and flow.

We begin by modeling the sprinkler adjoined to the main pipe as a tee-junction. According to [1], the head loss for fluid entering a tee-junction is given by

$$h_t = 1.8\frac{v_E^2}{2g} \tag{9}$$

Where v_E is the velocity of the ejected water. Because the water in the main pipe is moving at such a slow speed (0.212 m/s), we may assume that essentially all of the energy in the main pipe is stored in potential energy (i.e., pressure). Thus $h_f = \Delta P = P_i - P_f$. If we assume that all of the pressure is converted into kinetic energy at the sprinkler head, we have $P_f = \frac{1}{2}\rho v_E^2$. Plugging this into our equation above, we have

$$0.9v_E^2 = \frac{P_i}{\rho} - \frac{v_E^2}{2} \tag{10}$$

$$1.4v_E^2 = \frac{P_i}{\rho} \tag{11}$$

$$v_E = \left(\frac{P_i}{1.4\rho}\right)^{1/2} = 17.32 \text{ m/s} \tag{12}$$

We then use this to calculate the flow in the sprinkler. Whenever fluid flows through an orifice, the streamlines contract into an area smaller than that of the orifice [1]. Thus the effect radius of the orifice is reduced, by a factor known as the discharge coefficient. In lieu of experiment data, C_d is generally taken to be 0.62. Then we can calculate the flow through one sprinkler easily:

$$F = vA = (17.32)(\pi * (.003 * .62)^2) = 1.88 * 10^{-4} \text{ m}^3\text{/s} = 11.3 \text{ L/min} \tag{13}$$

4 Sprinkler head model

We next modeled the behavior of a single sprinkler. To do this we considered the sprinkler to be a point source at the origin ejecting water droplets uniformly in all directions. As discussed in [2], a typical rotary sprinkler can be considered to be ejecting two primary populations of water droplets: one small (0.5-1 mm) and one large (2-3 mm). These model the inner spray that flows from the left nozzle and the longer-range right nozzle respectively.

With the acknowledgment that many of the parameters are actually functions of the sprinkler design we used the following parameters:

- Initial distribution of radii: According to [3], a reasonable approximation to the distribution of radii of water droplets ejected from a sprinkler is

  $$V(D) = \frac{3}{D_0}\left(\frac{D}{D_0}\right)^5 \mathrm{e}^{(\mathrm{D}/\mathrm{D}_0)^3} \tag{14}$$

  where D_0 is the average diameter. The average water droplet diameter [4] for a typical sprinkler is 1.9 mm.

- Initial velocity distribution: We assume that for each nozzle the velocity distribution of the volume is roughly Gaussian, and several different standard deviations were used. Eventually a standard deviation of 5 m/s was chosen for the right nozzle as it provided a very uniform spray pattern.

- Initial angle distribution: The typical rotary sprinkler angle [2] is 30 degrees, small deviations from this were considered but not included in the final model.

The external forces acting on these water droplets during their flight are gravity and air resistance, where the force due to air resistance is:

$$F_f = C_f \rho A v^2, \tag{15}$$

and C_f is the coefficient of friction, which can be approximated [7] by

$$C_f = \frac{24}{\mathbf{R}} + \frac{6}{1+\sqrt{\mathbf{R}}} + 0.4. \tag{16}$$

This then gives us the following equation:

$$\frac{d(\vec{v})}{dt} = -g - F_f/m \tag{17}$$

where m is the mass of the water droplet. We then numerically integrated this equation using the weighted velocities and droplet sizes to find the resulting volume of water a point receives as a function of radial distance.

This gave us the water distribution for sprinklers at ground level and elevated 1 meter above the ground under various wind strengths as seen in figures (4.1) and (4.1). As expected, the greater wind strengths stretch the sprinkler distributions and increase the water density for the distribution facing into the wind.

Figure (4.3) [6] displays the water distribution for a sprinkler elevated by 1.7 meters. Notice that this distribution is two peaked, a feature that can be captured by our model with appropriate choice of parameters. We ran our complete algorithm using both the distributions from our own model and the distribution in figure (4.3).

5 Optimal Covering

Armed with the water distributions for our sprinklers we proceed to find the arrangement of sprinklers that most uniformly covers the field using two 80m pipes. With this assumption and some simplifying heuristics this problem was solved by enumeration of configurations.

5.1 Discretization

In order to make an enumeration based approach possible the field needed to be discretized. For this, the field was broken up into 80x30 one meter squares. A sprinkler was then represented by a grid, with each square meter grid cell having data about how many centimeters of water would be distributed in that cell over 1 hour.

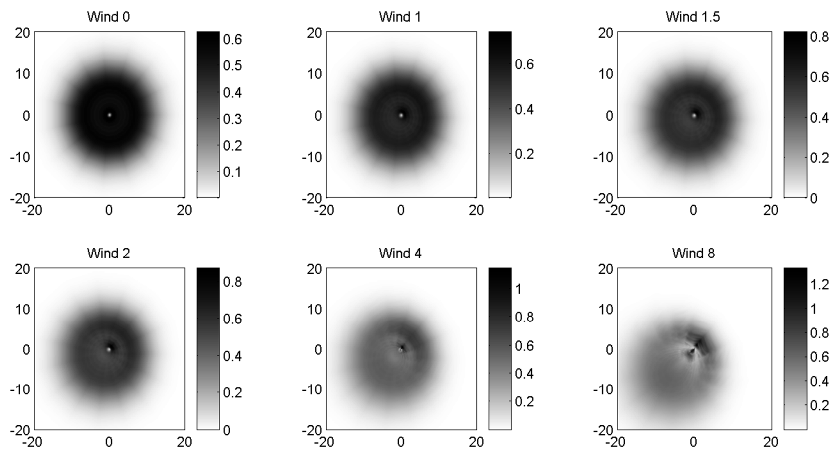

Figure 4.1: the sprinkler distributions for our sprinkler at ground level in winds up to 8 m/s where spatial units are in meters and waterfall is in cm/hour. This sprinkler setup was chosen for it's uniformity around the head.

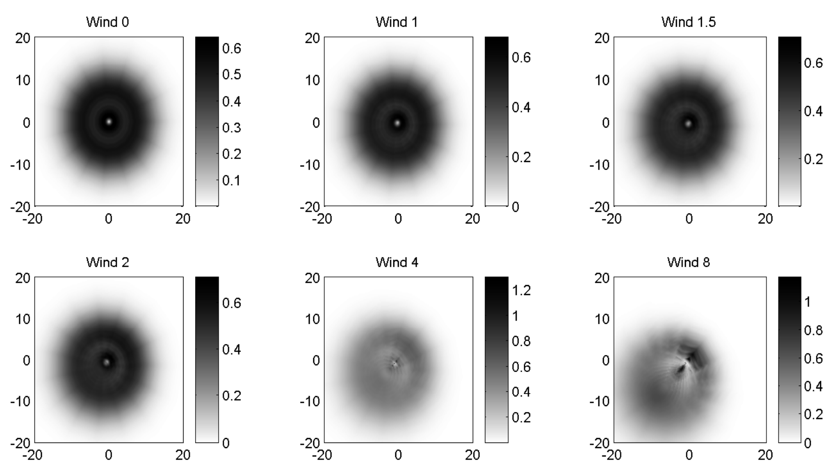

Figure 4.2: the sprinkler distributions for our sprinkler elevated by one meter in winds up to 8 m/s where spatial units are in meters and waterfall is in cm/hour

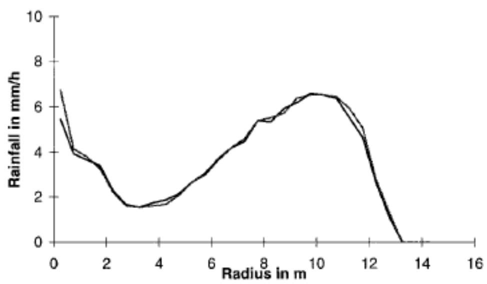

Figure 4.3: the sprinkler distribution found in [6] where pressure is 350kPa, the height is 1.7m and the orifice is approximately 4.5 mm. While unlike the figures (4.1) and (4.1) this distribution is two peaked and was initially avoided because of its uniformity.

5.2 Evaluation Function

The most natural way to express uniformity of water distribution is to simply consider statistical variance, and thus the lower the variance the better a field rank. This however gives a perfect rating to the setup of no sprinklers anywhere, and thus a cost had to be added in for leaving land unwatered. With our space F broken up into 1 meter squares the cost function for the field was

$$Cost[F] = \#SquaresWithoutWaterinF + Variance[F] \tag{18}$$

5.3 Search Space

It was assumed that in order to minimize the time necessary to maintain the sprinkler system, the best setup would be to arrange 4 pipes into 1 80 meter long pipe set (See Field Layout section). For the first stage of watering, this 80 meter pipe would be placed 10 meters from the edge of the field running lengthwise. For the next stage of the watering the pipe is moved over 10 meters towards the other side of the field. During this movement the pipe is broken up into 20 meter segments as to make it possible for 1 rancher to lift it easily without damaging the pipe. As the pipe is being disassembled, it is worthwhile to consider permutations the four pipe segments on the other side, as well as the option of flipping the pipe segment around. To narrow the search space, there were 10 different sprinkler segments considered for each 20 meter pipe segment, which are shown in (5.1).

5.4 Time

The bulk of the search time is spent evaluating the cost function for different sprinkler configurations. With 10 different options for each of the 4 pipe segments there are 10^4 possibilities for a pipe set's initial setup. Each one of those, depending on the number of unique pipes chosen, will have a certain number of permutations.

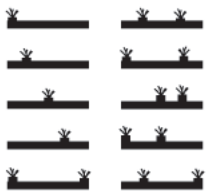

Figure 5.1: the ten pipe configurations considered when optimizing the uniform spray

Number Of Permutations

Number Of Permutations, x	Number Of Setups with x Permutations
1	9
4	360
6	270
12	4320
24	5040

Thus, there are

$$1 * 9 + 4 * 360 + 6 * 270 + 12 * 4320 + 24 * 5040 = 175869 \tag{19}$$

different permutations. The possibility of flipping pipes within a permutation adds another 2^4 options for each permutation, giving a total of 2813914 different cases to check. The evaluation function takes about .005 seconds to run, and generating a configuration takes about .001 seconds (with some pre-computation of sub-pieces), thus the algorithm takes about 4 hours of computation time per run. Run in parallel on 30 processors, this was an acceptable running time.

6 Results

Figure (6.1) displays the pipe that was found to water the field more uniformly with one moving than any other pipe. Since no pipe layout configuration without a moving could water every part of the field, this pipe configuration was the most efficient in terms of watering the entire field and minimizing the necessary maintenance time. Not only was this pipe found to be the best possible configuration for our first sprinkler at ground level, figure (4.1), but also for our sprinkler at height one meter, figure (4.2) and the sprinkler in figure (6.1)

Figure 6.1: Under two placements this pipe was found to adequately water the entire field

With that optimal arrangement of pipes, the distance between pipes was then varied to further decrease the variance in watering amounts. The best arrangement is having the pipe set 5 meters from each side of the field in succession. The below figure displays the distribution of water resulting from that arrangement.

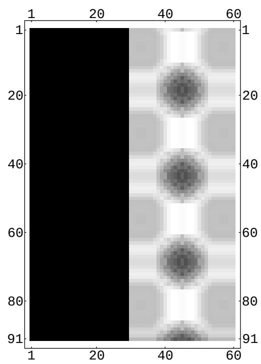

Figure 6.2: This shows the distribution of water from the two pipes when both have been on for equal amounts of time. The left side is an un-watered field given for comparison

With that distribution the most water received by any one area of the field is .6 centimeters per hour, while the minimum is .168 centimeters per hour. This allows the Rancher to leave the sprinkler on continuously and not exceed the maximum hourly flow rate. To achieve the minimum flow rate over a span of four days we must leave the sprinkler on for at least 12 hours on each side. This was calculated by noting that the minimum occured on an area that can only be wetted when the sprinkler is on one side of the field. Thus the sprinkler must be active on that side of the field at least $2/.168 \approx 12$ hours. Thus the optimal watering schedule would be to turn the sprinkler on in the evening, turn it off 12 hours later, wait a day, and then shift the sprinkler to the other side of the field and repeat. Doing this overnight is also optimal for dealing with evaporation[2].

7 Conclusion

Based on the short time required to upkeep hand-moved irrigation it presents a viable alternative to higher capital irrigation systems. This, when taken in conjunction with the versatility and mobility of hand-moved systems makes it the ideal solution for irrigating small to medium size fields with low initial capital. However, in terms of long-term investment the labor cost of a hand-moved irrigation system should be weighed against the initial saving associated with the hand-moved system.

References

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[7] Gerhart, Gross, and Hochstein. "Fundamentals of Fluid Mechanics", 2nd Ed. Reading, Mass.: Addison-Wesley, 1992.