Applying Voronoi Diagrams to the Redistricting Problem

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1 Introduction

Defining Congressional districts has long been a source of controversy in the United States. Since the district-drawers are chosen by those currently in power, the boundaries are often created to effect influence over future elections by grouping an unfavorable minority demographic with a favorable majority; this process is called *Gerrymandering*. It is common for districts to take on bizarre shapes, spanning slim sections of multiple cities and criss-crossing the countryside in a haphazard fashion. The only lawful restrictions on legislative boundaries stipulate that they contain equal populations, but the makeup of the districts is left entirely to the mapmakers.

In the United Kingdom and Canada, the districts are more compact. Their success in mitigating Gerrymandering can be attributed to the fact that they turn the task of boundary-drawing over to nonpartisan advisory panels. However, these independent commissions can take 2-3 years to finalize a new division plan, calling their effectiveness into question. It seems clear that the U.S. should establish similar unbiased commissions, yet make some effort to increase the efficiency of these groups. Accordingly, we present here a small toolbox of efficient tools to automatically establish legislative boundaries using a simple geometric construction.

1.1 Current Models

A majority of the methods currently available fall into two categories: ones that depend on a current division arrangement, most commonly counties, and ones that do not depend on current divisions. Most fall into the former category. By using current divisions, the problem is reduced to grouping these divisions in a desirable way using a multitude of mathematical procedures. Mehrotra et.al. uses graph partitioning theory to cluster counties to total population variation of around 2% [8]. Hess and Weaver use an iterative process to define population centroids, use integer programming to group counties into equally populated districts, and then they reiterate the process until the centriods reaches a limit [5]. Garfinkel and Nemhauser use iterative matrix operations to search for district combinations that are contiguous and compact [3]. Kaiser begins with the current districts and systematically swaps populations with adjacent districts [4]. All of these methods use counties since they divide the state into a relatively small number of sections. This is necessary since most of the mathematical tools they use become slow and imprecise when many elements are used [Ref]. This is the same as saying they become unusable in the limit when the state is divided into more continuous sections. Thus using small division, like zip codes which on average are 5 times smaller than a county in New York, becomes impractical.

The other category of methods is less common. Out of all our researched papers and documentation, there were only two methods that did not depend on current state divisions. Forrest's method continually divides a state into halves while maintaining population equality until the required number of districts is satisfied [4, 5]. Hale, Ransom and Ramsey create pie-shaped wedges about population centers. This creates homogeneous districts which all contain portions of a large city, suburbs, and less populated areas [4]. These approaches are noted to be the least biased since their only consideration is population equality. Also, they are straight forward to apply. However, they fail to consider geographical features or to keep cities within single districts.

1.2 Developing Our Approach

Since our goal is to create new methods that add to the diversity of available models from which a committee can choose, we should focus on creating district boundaries independently of current divisions. Not only has this approach not been explored to its fullest, but it isn't obvious why counties are a good beginning point for a model. Counties are created in the same arbitrary way as district, so they stand to contain implicit biases. There is no immediate reason to believe these biases will be nulled when different combinations of counties are formed. Also, counties are relatively large in comparison to a district. In NY, districts contain on average around 4 counties. Some districts only contain two counties. Many of the division dependent models end up to relaxing their boundaries from county lines in order to maintain equal populations, which not only weakens the initial assumption of using county divisions but also allows for gerrymandering if this relaxation method is not clearly defined.

Treating the problem as continuous doesn't lead to any specific type of approach. It gives us a lot of freedom but at the same time implies we can impose more conditions. If the Forrest and Hale et.al. methods are any indication, we should focus on keeping cities within districts and introduce geographical considerations. (Note that these conditions do not have to be considered if we treat the problem discretely, like in the first method, because those are implicitly dependent on prominent geographical features).

`Goal-` create a method for redistricting a state by treating the state continuously. We require the final districts to contain equal populations and be contiguous. Additionally, the districts should be as simple as possible (see sec. 2 for a definition of simple).

2 Notation and Definitions

- **contiguous:** a set R is contiguous if it is pathwise-connected
- **compactness:**we would like the definition of compactness to be intuitive. One way to look at compactness is the ratio of the area of a bounded region to the square of its perimeter. In other words

$$C_R = \frac{A_R}{p_R^2} = \frac{1}{4\pi}\mathcal{Q}$$

  where C_R is the compactness of region R, A_R is the area, p_R is the perimeter and $\mathcal{Q}$ is the isoperimetric quotient.
- **simple:** any region that is contiguous, compact and convex.
- **Voronoi diagrams:** a partition of the plane with respect to n nodes in the plane such that points in the plane are in the same region of a node if they are closer to that node than to any other point (for a detailed description, ref 3.1)

- **generator point:** a node of a Voronoi diagram
- **degeneracy:** number of districts represented by one generator point
- **Voronoiesque diagram:** variation of the Voronoi diagram based on equal masses of the regions (ref 3.3)
- **population center:** region of high population density

3 Theoretical Evaluation of our Model

How we analyze our results is a tricky affair since there is disagreement in the redistricting literature on key issues. **Population equality** is the most well defined. By law, populations within districts have to be the same to within a few percent of each other. No specific percentage is given, but can be assumed to be around 5%.

There is also the requirement for **contiguity**. Districts need to be path-wise connected so that one part of a district cannot be on one side of the state and the other part on the other end of the state. This requirement is meant to maintain locality and community within districts. It does not, however, restrict islands districts from including islands if the island's population is below the required population level.

Finally, there is a desire for the districts to be, in one word, **simple**. There is little to no agreement on this characteristic, and the most common terminology for this is *compact*. Taylor defines simple as a measure of divergence from compactness due to indentation of the boundary and gives an equation relating the nonreflexive and reflexive interior angles of a region's boundary [9]. Young provides seven more measures of compactness. The *Roeck* test is a ratio of the area of the largest inscribable circle in a region to the area of that region. The *Schwartzberg* test takes ratio between the adjusted perimeter of a region to a the perimeter of a circle whose area is the same as the area of the region. The *moment of inertia* test measures relative compactness by comparing "moments of inertia" of different district arrangements. The *Boyce-Clark* test compares the difference between points on a district's boundary and the center of mass of that district, where zero deviation of these differences is most desirable. The *perimeter test* compares different district arrangements buy computing the total perimeter of each. Finally, there is the *visual* test. This test decides simplicity based on intuition [11].

Young notes that "a measure [of compactness] only indicates when a plan is more compact than another"[11]. Thus, *not only is there no consensus on how to analyze redistricting proposals, it is difficult to compare them.*

Finally, we remark that the above list only considers the shape of a district. There is no mention of any other possibly relevant features. It doesn't consider how well populations are distributed or how well the new district boundaries conform with other boundaries, like counties or zip codes. Even with this short list, it is clear that we are not in a position to define a rigorous definition of simplicity. What we can do, however, is identify features of our proposed districts which are simple and which are not. This is in line with our goal defined in sec. 1.2, since this list can be provided to a districting commission from which they will decide how relevant those simple features are.

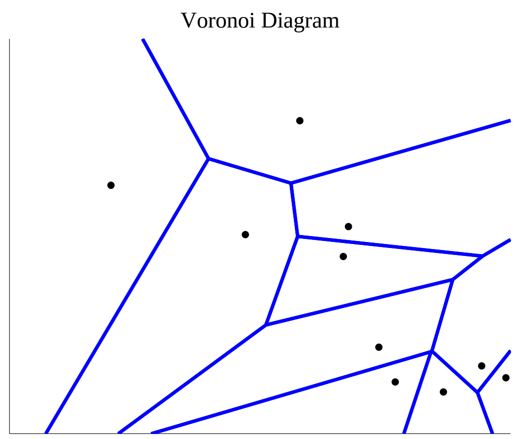


Figure 1: Illustration of Voronoi diagram generated with Euclidean metric. Note the compactness and simple boundaries of the regions.

We do not define *simple*, we merely state the features of our model that can be considered simple.

4 Method Description

Our approach weighs heavily on using Voronoi diagrams. We begin with a definition, its features, and motivate its application to the redistricting.

4.1 Voronoi Diagrams

A Voronoi diagram is a set of Voronoi polygons with respect to n generator points contained in the plane. Each generator p_i is contained within a Voronoi polygon $V(p_i)$ with the following property:

$$V(p_i) = \{q | d(p_i, q) \leq d(p_j, q),\ i \neq j\} \text{ where } d(x, y) \text{ is the distance from point } x \text{ to } y$$

That is, the set of all such q is the set of points closer to p_i than to any other p_j. Then the diagram is given by (see fig 1)

$$\mathbf{V} = \{V(p_1), \ldots, V(p_n)\}$$

Note that there is no assumption on the metric we use. There are many metric we could use, but we only consider the three common metrics:

- Euclidean Metric: $d(p, q) = \sqrt{(x_p - x_q)^2 + (y_p - y_q)^2}$
- Manhattan Metric: $d(p, q) = |x_p - x_q| + |y_p - y_q|$
- Uniform Metric: $d(p, q) = \max\{|x_p - x_q|, |y_p - y_q|\}$

4.1.1 Useful Features of Voronoi Diagrams

Here is a summary of relevant properties:

- The Voronoi diagram for a given set of generator points is unique
- The nearest generator point of p_i determines an edge of $V(p_i)$
- The polygonal lines of a Voronoi polygon do not intersect the generator points.
- When working in the Euclidean metric, all regions are convex, while at the same time the generator points are spaced equally apart pairwise.
- When using the Manhattan metric, convexity is lost but each Voronoi polygon is diamond-shaped with respect to the generator points the polygon embodies.
- There are no restrictions on the type of line segments formed in the Euclidean metric whereas the Manhattan metric is only capable of creating borders that are either vertical, horizontal, or angled at 45 or 135 degrees.

These properties are important for our model. The first property tells us that regardless of how we choose our generator points we generate unique regions. This is good when considering the politics of Gerrymandering. The second property implies that each region is defined in terms of the surrounding generator points while in turn, each region is relatively compact. **These features of Voronoi diagrams effectively satisfy two out of the three criteria for partitioning a region: contiguity and simplicity**.

4.2 Voronoiesque Diagrams

Our next method differs from including Voronoi diagrams. Nevertheless we will call these diagrams Voronoiesque because they are similar in structure to actual Voronoi diagrams. An alternative way for Voronoi diagrams as described in section 3.1 to be generated is by letting each generator point grow radially outward (in whatever metric that is being considered) at the same rate. As the regions intersect, they will form the boundary lines for the regions. Voronoiesque diagrams are defined similarly with one exception: for some real-valued function f defined at every $(x, y) \in \mathbf{V}$ we have after each iteration of growth

$$\left| \int_{\mathcal{V}_i} f(x,y)dA - \int_{\mathcal{V}_j} f(x,y)dA \right| < \epsilon \text{ for } i \neq j \text{ and for sufficiently small } \epsilon > 0$$

where each $\mathcal{V}_i$ is a Voronoiesque region such that after a sufficient number of iterations, $\bigcup_{i=1}^{n} \mathcal{V}_i = \mathbf{V}$ (Note that ϵ notation is used because there is no theory presented that guarantees the possibility of the left hand side of the above expression being zero at every iteration). What's useful about Voronoiesque diagrams are their requirement of approximate mass equality between regions after each iteration. The final consideration is population equality by finding locations for generator points.

4.3 Determining Generator Points Using Population Density Distributions

In order to create regions using Voronoi diagrams, we need to define the locations of the generator points. This is our only degree of freedom since generator points generate unique Voronoi regions. The most intuitive approach is to choose our generator points to be at the population distribution centers. As mentioned in section 4.1.1, polygonal lines produced by generator points do not intersect the generator points. Thus the generator points we choose keep larger cities within within the generated boundaries. However, this has a problem. some cities need to be divided to maintain population equality. New York City is a perfect example. It contains enough people to hold 13 districts.

Consideration for Large Cities

When we are determine generator point locations, we weigh each point by the number of districts that city peak and the peaks in close proximity has to maintain. For all the areas that need to be divided into $n > 1$ districts, we give it the generator point a weight of n and consider that to be a single point with a *degeneracy* of n. We assign generator points to population peaks until the sum of all the points and their respective degeneracies is equal to the number of state representatives. In other words, until:

$$\sum_{\text{all generator pts.}} \text{degeneracy of generator pt.} = \#\text{ state representatives}$$

Once the generator points are defined, we create a Voronoi diagram. Then we look at the polygons containing generators with degeneracies greater than one, create generator points in the same way as before, and then create Voronoi polygons within that region.

As we will see when we apply our model to New York, this method works well. It should be noted, though, that this is not the only way to define the location of generator points, but it is a very good first approach.

4.4 Creating Regions using Voronoi and Voronoiesque Methods

We have two methods of creating diagrams. Applying the Voronoi method is straight forward. The resulting Voronoi diagram divides the region into polygonal districts. We create districts for all three metrics: Euclidean metric, Manhattan metric, and uniform metric.

The Voronoiesque method is a little bit more complicated. We use population density as our function function $f(x, y)$. So with each iteration of region expansion, the population stays constant. Once these expanding regions intersect, they form the boundaries of our districts. We create districts for all three metrics.

5 Redistricting in New York State

At this point, we have described a general procedure for generating political districts with Voronoi diagrams which seems effective, in theory. However, since every theory must eventually be tested on a concrete example, we turn our attention to the task of redistricting a particular state. We choose to redistrict New York since it seems to pose a great difficulty to any algorithm aiming to partition a state into geometrically simple regions with equal population. In particular, New York

- **has regions with large population density**,
- **has regions with constrained geography**,
- and **must be divided into many (29) regions**.

We begin by explaining our method for choosing generator points at population centers, since these points will uniquely determine a Voronoi diagram for the state. Then we describe several methods for generating Voronoi and Voronoiesque diagrams from these points and present the corresponding results. Finally we discuss how to use these diagrams to create actual political districts for New York state.

5.1 Population Density Map

To apply our Voronoi diagram methods to New York, we must first obtain an approximate population density map of the state. The U.S. Census Bureau maintains a database [2] which contains census tract-level population statistics; when combined with boundary data [1] for each tract, it's possible to generate a density map with a resolution no coarser than 8,000 people per region [7]. Unfortunately, our limited experience with the Census Bureau's data format prevented us from accessing this data directly, and we contented ourselves with a 792-by-660 pixel approximation to the population density map [6].

We loaded this raster image into MATLAB and generated a surface plot where height represented population density at each point. To remove artifacts introduced by using a coarse lattice representation for finely-distributed data, we applied a 6-pixel moving average filter to the density map. The resulting population density is shown in fig. 2.

5.2 Limitations of the Image-Based Density Map

The population density image we used yielded a density value for every third of a square mile from the following set (measured in people per square mile):

$$\{0,\ 10,\ 25,\ 50,\ 100,\ 250,\ 500,\ 1000,\ 2500,\ 5000\}\,.$$

This provides a decent approximation for regions with a density smaller than $5,000\ people/sq.mi.$ However, since New York City's average population density is $26,403\ people/sq.mi.$ [10], the approximation will break down at large population centers. We will make note in the subsequent analysis whenever this limitation has a profound effect on our results.

5.3 Selecting Generator Points

Our criteria for redistricting the state stipulates that the regions we generate must contain equal populations. New York state must be divided into 29 congressional districts to support its share of representatives, so each region must contain $\approx 3.45\%$ of the state's

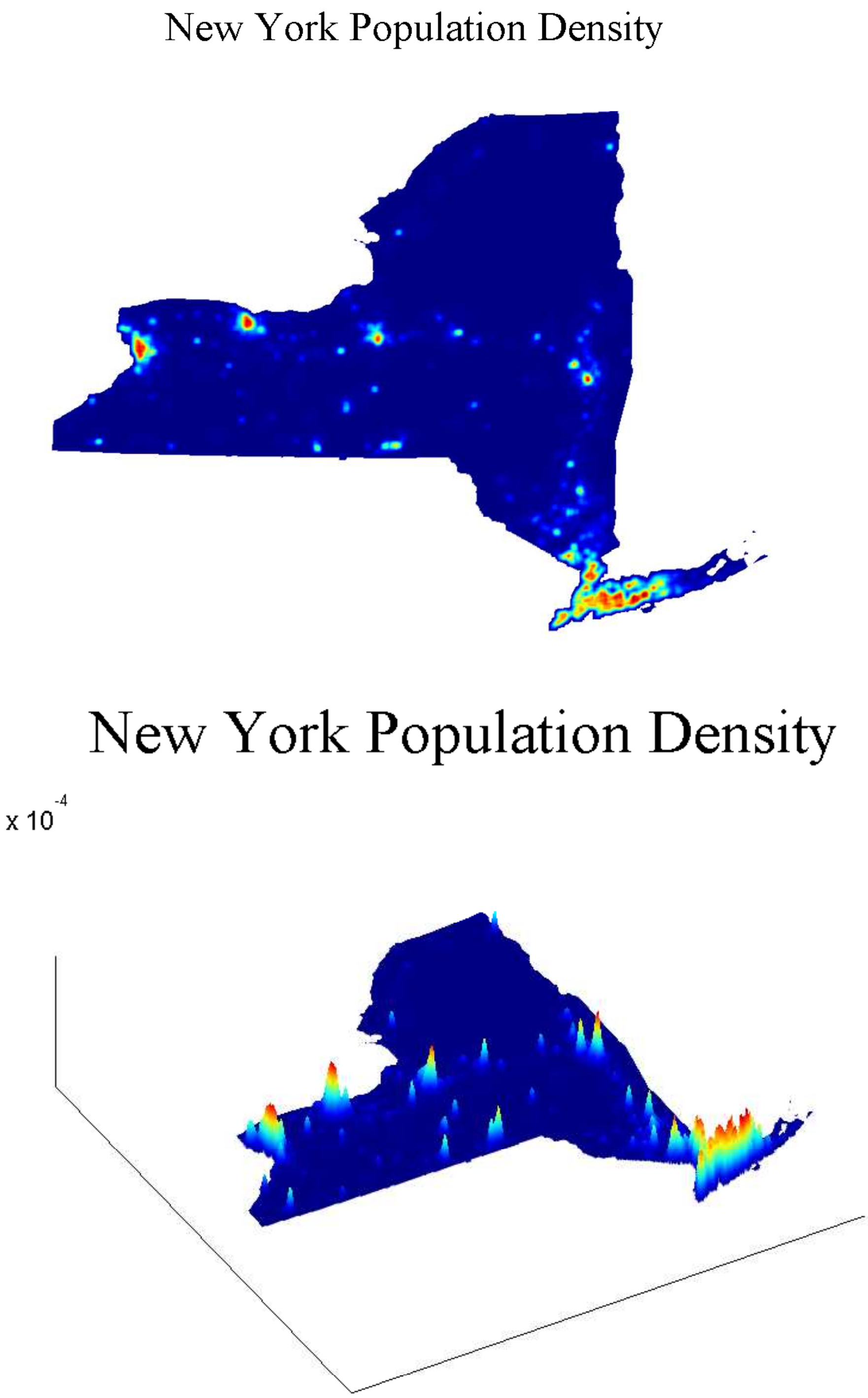


Figure 2: New York State population density map. Obtained from a 792-by-660 pixel raster image; color and height indicate the relative population density at each point.

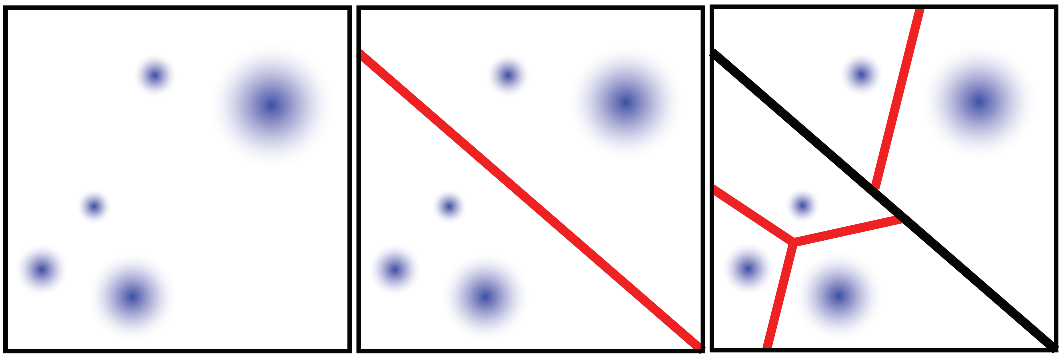


Figure 3: Left-to-Right: illustration of the subdivision process for particularly dense regions. Perspective: overhead view of population density map.

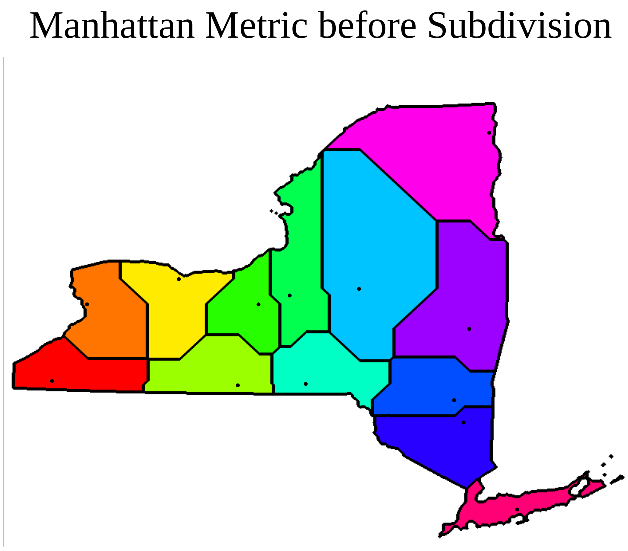
population. Since a state's population is concentrated primarily in a small number of cities, we use local maxima of the population density map as candidates for generator points.

If we were to simply choose the highest 29 peaks from the population density map as our set of generator points, the resulting set would be contained entirely in the largest population centers and would not reflect the overall population distribution. Instead, we initially select one peak in each population center, generate a Voronoi diagram for the state from these points, then return to the particularly dense regions of this diagram and subdivide them. We create the subdivision of a region in a manner similar to how we generated the initial partition, except this time we place no restrictions on how close the generator points may be and simply pick from the density maxima. See fig. 3 for an illustration of the decomposition before and after subdivision.

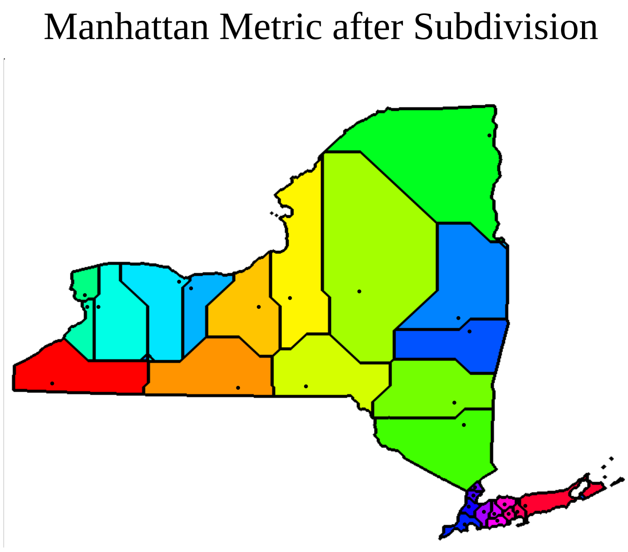
Based on our density data for New York state, we subdivided the region around New York City into 12 subregions, Buffalo into 3 subregions, and Rochester and Albany into 2 subregions. Note that this roughly corresponds to the current allocation, where New York City receives 14 districts, Buffalo gets 3, and Rochester and Albany both get roughly 2. Here, New York City's population is underestimated since the average density there far exceeds our data's density range. With a more detailed data set, our method would have called for the correct number of subdivisions.

5.4 Applied Voronoi Diagrams

The simplest method we consider for generating congressional districts is to simply generate the discrete Voronoi diagram from a set of generator points. We achieve this by iteratively 'growing' regions outward from the generator points on a rectangular lattice of pixels (the pixel lattice is taken from our population density map); see fig. 5. A region's growth is limited at each step by its radius in a certain metric; we considered the Euclidean, Manhattan, and uniform metrics. Once the initial diagram has been created, a new set of generator points for dense regions are chosen and those regions are subdivided using the same method. Unrefined decompositions can be seen in fig. 6.



(a) Diagram using initial generator points.



(b) Diagram after subdivision of dense regions (New York City, Buffalo, Rochester, and Albany).

Figure 4: Two stages of the creation of a Voronoi diagram for legislative districts.

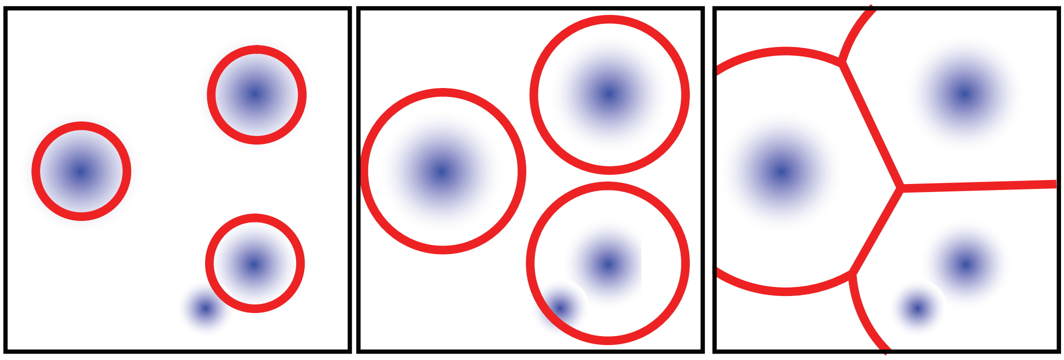
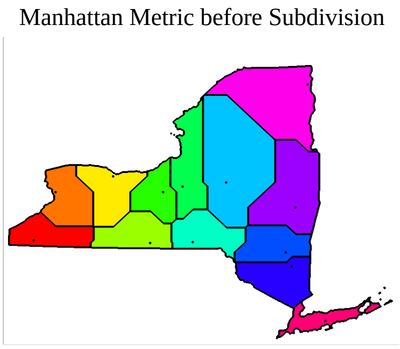
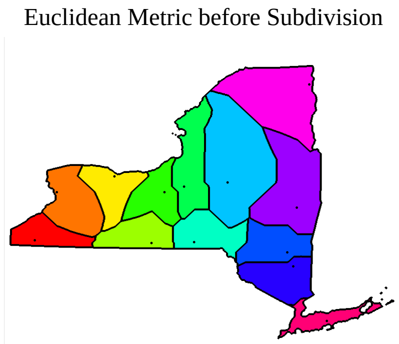


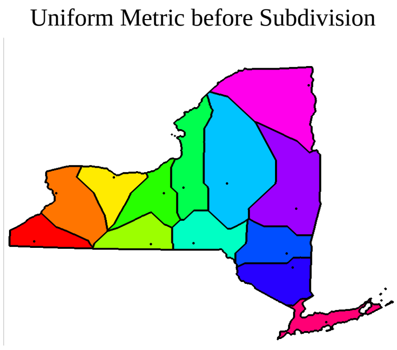
Figure 5: Left-to-Right: illustration of the process of 'growing' a Voronoi diagram. Perspective: overhead view of population density map.



(a) Average Population $= (3.5 \pm 2.2)\%$.



(b) Average Population $= (3.7 \pm 2.6)\%$.



(c) Average Population $= (3.7 \pm 2.6)\%$.

Figure 6: Voronoi diagrams generated with three distance metrics before subdivision of densely populated regions.

Region #	Population %	Region #	Population %	Region #	Population %
1	3.03	2	3.02	3	6.15
4	3.51	5	3.43	6	3.45
7	3.43	8	3.22	9	3.50
10	3.19	11	2.78	12	3.33
13	3.21	14	3.00	15	3.43
16	3.38	17	4.43	18	4.76
19	3.24	20	3.12	21	2.97
22	3.17	23	3.17	24	3.21
25	2.44	26	2.12	27	3.66
28	3.79	29	3.71		

Table 1: Population Fraction in each Legislative District

Each metric produces a relatively simple decomposition of the state, though the Manhattan metric has simpler boundaries and yields a slightly smaller population variance between regions.

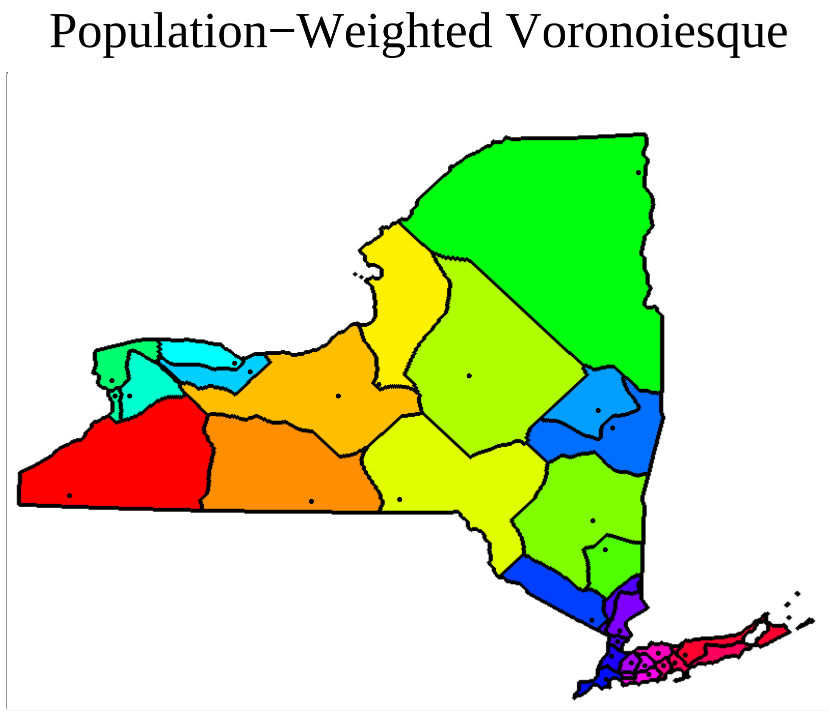
5.5 Applied Voronoiesque Diagrams

Though our simple Voronoi diagrams produced simple regions with a population mean near the desired value, the population variance between regions is enormous. In this sense, the simple Voronoi decomposition doesn't meet one of the main parts of our redistricting goal. However, the Voronoi regions are so simple that we prefer to augment this method with population weights rather than abandon it entirely.

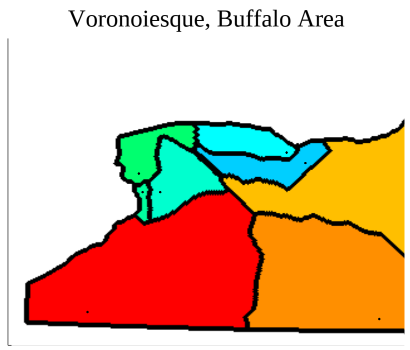
Fig. 7 shows the result of this decomposition, along with exploded views of the two regions which were subdivided more than twice in the refinement stage of the diagram generation. The population contained in each region is summarized in table 1.

5.6 Precisely Defining Boundary Lines

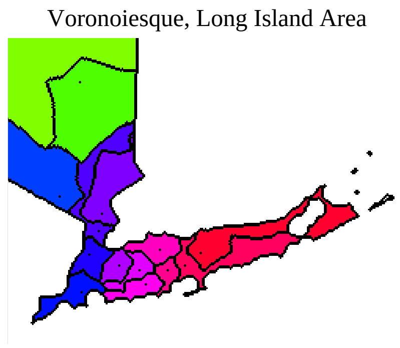
It is impractical to use the regions generated here directly to divide a state, if only because no care was taken to ensure the boundaries didn't slice a house in half. However, given the scale at which the Voronoi and Voronoiesque diagrams were drawn, it seems reasonable to assume that their boundaries could be modified to trace existing boundaries—like county lines, ZIP codes, or city streets—without changing their general shape or average population appreciably. In particular, the average area of a ZIP code in New York state is $\approx 10\ sq.mi.$ and there are roughly 200 city blocks per square mile in Manhattan, while the minimum size of one of our Voronoi regions is 73 $sq.mi.$ and the average size is $\approx 2,000\ sq.mi.$. Therefore it seems reasonable that existing boundaries could be used to implement very close approximations to our Voronoi diagrams.



(a)



(b) Exploded view of regions around Buffalo.



(c) Exploded view of regions around Long Island.

Figure 7: Population-weighted Voronoiesque diagram for New York state. Average population per region = $(3.34 \pm 0.74)\%$.

6 Analysis

6.1 New York State Results

We turn now to a discussion of how well our results from the previous section meet our original specification for redistricting. In terms of simplicity of generated districts, our Voronoi diagram method is a clear winner, particularly when applied with the Manhattan metric: the generated regions are contiguous and compact while their boundaries, being unions of line segments, are about the simplest that could be expected. However, this method falls short in achieving equal population distribution among the regions, since the variance in the average population per region is on the order of the average population itself.

As may be expected in any sort of high-dimensional optimization problem, there is an essential tradeoff in this problem between the simplicity of the legislative districts and their respective populations. Accordingly, when we modify the Voronoi diagram method to generate population-weighted Voronoiesque regions, we cut the population variance by a factor of four—from $\pm 2.8\%$ to $\pm 0.7\%$—while suffering a small loss in the simplicity of the resulting regions. In particular, regions in the Voronoiesque diagrams are generally less compact and their boundaries are more complicated than their Voronoi diagram counterparts, though contiguity is still maintained.

Finally, we noted in the previous section that any actual implementation of a diagram generated from either of our methods would have to make small, localized modifications to ensure the district boundaries make sense from a practical perspective. Though this would appear to open the door for the same sort of politically-biased district manipulations our methods were aiming to avoid in the first place, we think the size of the necessary deviations—on the order of miles—is small enough when compared to the size of a Voronoi or Voronoiesque region—on the order of tens or hundreds of miles—to make the net effect of these variations insignificant. Therefore, though we have provided only a first-order approximation to the congressional districts, we have left little room for Gerrymandering to occur.

6.2 General Results

We already know how well our results worked for New York. How effective is our method in general? We examine the results for an arbitrary state including worst case scenarios for each crtiteria.

Population Equality

The largest problem with this requirement occurs when we try to make regions too simple. Typically, our first method has the most room for error here. If a state has a series of high population density peaks with a relatively uniform decrease in population density extending away from each peak, then the regions will differ quite a bit. This is becuase in this situation, ratios of populations are then roughly equal to the ratios of areas between regions. However, our final method focuses primarily on population so equality is much easier to regulate here.

Contiguity

Contiguity prolems arise often if the state itself has little compactness, like Florida, or if the state has some sort of sound like Washington. The first two methods focus more just population density without really acknowledging the boundaries of the state itself. So it's possible for one region to be separated by some geographic obstruction like a body of water or a mountain range. Again the final method fixes this by growing in increments, this allows for state boundaries to be defined. Then regions wouldn't grow over but around those obstacles.

Compactness

Unfortunately, the final method doesn't do everything, it is the least likely candidate for generating compact regions. The first two are most successful in this area. The first method creates all convex regions. Though the second can't guarantee convexity, its form is similar in shape and size to the first. Furthermore, one nice property of the generated regions from the first method is that there is a way to make slight adjustments to the boundaries while still maintaining convexity (see sec. 7.1) This is good for taking population shifts across districts into account between redestricting periods.

7 Improving the Method

Now that the problem areas have been defined, we offer some ways to reduce the effect of these problems.

7.1 Boundary Refinement

Consider the Voronoi diagram method. We know this approach is good at generating convex districts but not as successful at maintaining population equality. One such method that helps is vertex repositioning. Notice that adjacent districts generated by this method all share a common vertex common to all three boundaries. From this vertex extends a finite number of line segments that partially define the boundaries of these adjacent regions. Connecting the endpoints of these segments yields a polygon. Now we are free to move the common vertex anywhere in the interior of this polygon while still maintaining convexity. With this we can redraw boundaries between regions that are significantly different in population size and in doing so help equalize each of the regions.
There are also ways to adjust population inequality in the Voronoiesque method. Looking at the region with the lowest population, systematically increase the area of the low-population regions while decreasing the area of the neighboring high-population regions.

7.2 Geographic Obstacles

Our method doesn't implement geographic areas such as rivers, mountains, canyons, and other prominent features. What would greatly improve our Voronoiesque method would be to construct an algorithm that specifically addresses and deals with these obstacles. These

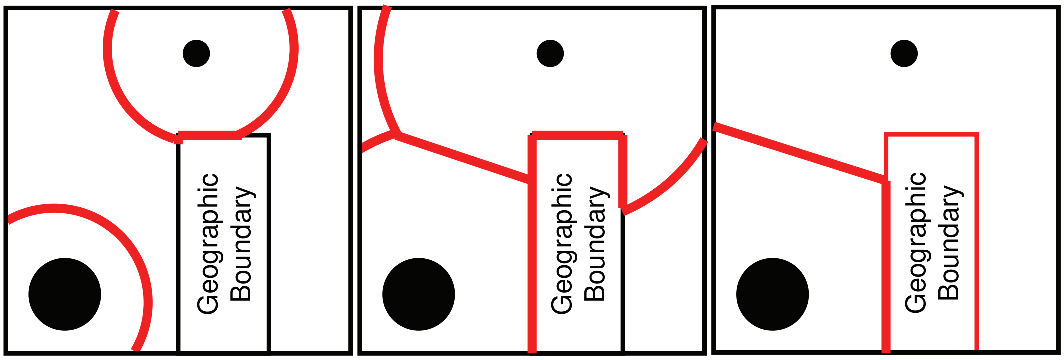


Figure 8: Illustration of Voronoi diagram generation which takes geographic obstacles into account.

can be defined in the same way as the state boundaries. However, this makes contiguity that much more of an issue. An illustration of this method is shown in fig. 8.

8 Bulletin to the Votors of the State of New York

READ ON FOR IMPORTANT INFORMATION REGARDING YOUR REPRESENTATIVE GOVERNMENT

Authorities within your state's government recently realized that during reapportionment—the process by which your state's number of congressional representatives changes—the incumbent political leaders tend to *Gerrymander* the boundaries of congressional districts, redrawing them to influence future elections in their favor. As this can undermine equal representation for all citizens, the State of New York commissioned an interdisciplinary team of mathematicians and engineers to create an objective procedure for redistricting that can be applied in the future to prevent partisan influence over congressional district boundaries.

The team came to the conclusion that to be fair to all, congressional districts should:

- **be connected**,
- **contain equal populations**,
- **be as compact as possible**, and
- **not unnecessarily subdivide large cities**.

Accordingly, they created a simple method for generating districts that meet these criteria.

The method is based on a geometrical structure known as a *Voronoi diagram*, which describes a partition of your state into compact, connected regions generated from a set of initial points; see figure 1 for an example. Since the regions are supposed to envelop equal populations, the initial points were chosen at major population centers (like New

York City, Buffalo, Rochester, and Albany, among others). The regions are then ‘grown’ out from these population centers as in figure 5 until the entire state is covered.

To ensure the districts end up with roughly equal populations, a regions’ growth is limited by the population contained within it. This results in a final diagram which has connected, compact regions with small population variation. In other words, diagrams generated with this method fulfill the guidelines for creating fair legislative districts.

The new district diagram is illustrated in figure 7. This diagram is composed of 29 distinct congressional districts, each of which contains close to 3.4% of the total state population. But more important than the precise population contained in each region is the fact that the districts were generated objectively by a computerized method, so *partisan politics play no role in the result.* This ensures that the next time boundary lines are drawn, they will provide an impartial partition of our state’s population, with no room for Gerrymandering.

9 Conclusion

There are many methods in existence for drawing district boundaries. Furthermore each is mostly successful in what is sets out to do. However, all of these seem to operate on using county lines as a guideline. Our model differs in that initially we only consider population and simplicity, while leaving room for county, property, and geographic considerations later.

We presented a toolbox of several different methods of generating district boundaries. Keeping in mind that hypothetically, there exist scenarios where a single method is most desirable, we found that for New York especially, our Voronoiesque method was most accurate. What’s particularly attractive about all the methods we presented is that not only is the underlying reasoning accessible to most individuals, but also that when properly implemented, the actual generation process takes a few seconds or less.

Our results are based on census data for the State of New York that was sufficient for demonstrating the success of our models. The problem areas for our model arise from ideal and therefore unrealistic situations such as uniform population distribution. But even in this case models like the Forrest method can easily generate perfect districts. Our method is applicable to other states and population dispersions.

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