Lost and Found: Mathematically Locating Ocean Downed Aircraft

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Abstract

Given the vast size of the Earth's oceans, and the dynamics of their currents, any attempt to give the exact location of a downed aircraft is infinitely daunting. Instead of attempting the exact, what we propose in this paper is an approach to the problem that maximizes search efficiency. Our approach will be developed over several steps: 1) develop the **disappearance interval** and **search radius**, 2) model the trajectory of the aircraft while still in the air using **scenario and probability analysis**, 3) model the **effects of the ocean's currents on the fuselage** at the crash site, and 4) apply Bayesian search methods to maximize **search efficiency**. Using the aircraft's velocity, trajectory, and location at time of last contact, we apply a modified version of the equations of motion to determine the possible crash points within a bounded region. Working within this region, we apply different probability distributions to the aircraft's location under varying scenarios such as mechanical failure or bad weather conditions. We then analyze the effects of ocean currents on the fuselage to determine a maximum drift. Accordingly, our model will give us a series of estimates of the aircraft's resting location on the seafloor. Each of these resting locations will be defined in a search area and assigned a probability so that Bayesian search theory can be applied. Finally, we will address how to test our model for accuracy and how our model might be improved by available technology, such as advanced satellite tracking.

Contents

1 Introduction

Given recent events concerning ocean downed aircraft, we understand that efficient methods and models are needed to aid search and rescue efforts. The sheer size of the Earth's oceans render any search and rescue efforts formidable and so we will utilize well-developed mathematical principles to aid in these efforts.

In this paper we present a model for locating aircraft lost at sea and presumed to be downed. Our model takes into account critical factors such as location, trajectory, and velocity at point of last contact. Additional factors considered in the model are aircraft mass as well as prevailing weather and ocean currents. This information is used in our model to determine the most likely crash and resting sites of the aircraft. The model maximizes search efficiency by minimizing the time it takes to locate the aircraft.

1.1 Outline of Our Approach

The beginning of our paper will focus on developing the necessary framework for the model to work. Here we consider the reasons which may have caused the aircraft to go down. Reasons due to mechanical failure or severe weather have different implications on pilot reaction. Additionally, we consider the possible scenarios once the aircraft hits the ocean; it may sink or may drift for an undetermined amount of time before sinking. These scenarios are all considered throughout the development of the model. Our priorities are as follows:

- **Develop the model: Search Efficiency**
  Throughout the development of our model, we use a modified versions of the equations of motion, probability distribution fitting, and Bayesian search methods. The motivating idea is to create efficient search techniques while working against a clock. We identify the most probable crash sites and a maximum search radius.
- **Probability Density Fitting**
  Different distress scenarios imply different pilot reactions. We identify the most probable causes of an aircraft crash to be: extreme weather conditions or mechanical/human failure. Under this scenario analysis, we fit a probability density function centered on the most probable crash sites which are considered to be random variables.
- **Bayesian Search Method**
  Given a bounded search region, we establish disjoint subregions each with an assigned probability of containing the fuselage. Within the sub regions, another probability of locating the plane is es-

tablished and is based on factors such as water depth, search equipment, and technology. Using Bayesian statistics, the results are a decreased probability in areas searched and increased probability in the other subregions. This may seem like common sense, but having an organized and systematic search method based on probability theory will aid any ocean search and rescue effort. Bayesian search methods were successfully employed in the location of Air France Flight 447 [2].

- **Analysis of Results: Suggestions for Improvement and Implementation**
  Testing the model against unavailable real data proved difficult. What we are confident in is the approach to the problem and the modeling that we propose on maximizing the search efficiency. We offer suggestions for further improvement and implementation of the model through simulation in ocean search and rescue efforts.

1.2 Assumptions

Due to the complicated nature of locating missing aircraft, we will introduce a few assumptions to our model.

- Any aircraft must communicate with air traffic control (ATC) every thirty minutes. With current technologies, communication between aircraft and ATC is more frequent for many aircraft and thirty minute intervals is standard for other aircraft following protocol. The thirty minute limit assumption gives a definite time frame to work within which the aircraft disappeared. Our model can be applied similarly in situations where radar contact is lost but the aircraft continues to send basic information (velocity, altitude, latitude and longitude) to satellite receivers (as many modern aircraft do).
- The aircraft has crashed due to outside forces (weather) or a mechanical error of some kind. Our model was not designed to analyze a hijacking or intentional crash off-course.
- When confronted with a storm, a pilot will fly above or through the storm. They will not attempt to fly around the storm unless given authorized permission from ATC. This is in line with protocol; a pilot will not alter course unless he is authorized to do so by ATC.
- The aircraft is not sending out any signals or information regarding its location. If this were the case, our model would not be necessary to locate the airplane.
- The aircraft will sustain damage upon crashing. Specifically, we will assume the fuselage has separated from the wings of the

aircraft. An aircraft is highly unlikely to undergo a major crash and lose communication abilities if it has not sustained major damage in some way.

1.3 Definition of Terms

- **Disappearance Interval:** Time in between last point of contact and scheduled point of contact. If scheduled contact is not made, then the disappearance interval is the maximum of the time elapsed between points of contact. Our assumption is that the disappearance interval is thirty minutes.
- **Search Efficiency:** Governing search principle; constant update of high probability search areas through Bayesian analysis.
- **Mayday Indicator:** The location that the aircraft begins to lose control in the sky.
- **Crash Point:** The location that the aircraft makes contact with the water.
- **Resting Location:** The final location of the aircraft once it reaches the ocean floor.

2 Probability Density of Mayday Indicators

In order to apply the Bayesian search method, we first develop a probability density function (PDF) that will aid in prioritizing search areas within the established radius. We consider the two scenarios that motivate our model development whose characteristics prescribe distinct PDFs.

2.1 Storm Scenario

Given our assumption that a pilot will fly above or into the middle of a storm, we consider the probable location of the craft as it is being taken down by the storm. In this sense, the mayday indicators are best modeled by a positively skewed log normal density function [10]:

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} x \exp\left(-\frac{[\ln(x)-\mu]^2}{2\sigma^2}\right) \tag{1}$$

with the mean, mode, variance and skewness defined:

$$\text{Mean:} \quad e^{\mu+\sigma^2/2} \tag{2}$$

$$\text{Mode:} \quad e^{\mu-\sigma^2} \tag{3}$$

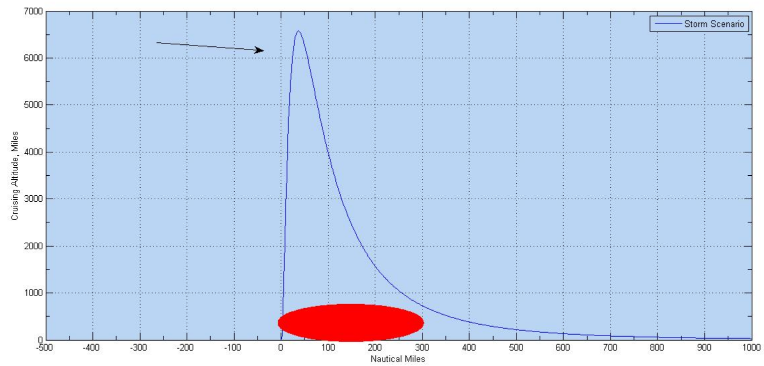

Figure 1: A sample PDF in a storm scenario with the aircraft represented by the black arrow. The red area highlights the highest probability locations for mayday indicators and thus the highest density of mayday indicators

$$\text{Variance:} \quad (e^{\sigma^2} - 1)e^{2\mu+\sigma^2} \tag{4}$$

$$\text{Skewness:} \quad (e^{\sigma^2} + 2)\sqrt{e^{\sigma^2} - 1} \tag{5}$$

where μ is the location parameter.

We will define the mode of our PDF to be the center of the storm. We have chosen this designation as it is unlikely the aircraft will immediately fall out of the sky once entering the storm (even though it is possible) but will instead fly some undetermined distance before crashing.

2.2 Mechanical Failure Scenario

The event of mechanical failure over the ocean expands the probable search area. Under this scenario the location of the aircraft is best modeled by a normal distribution with a higher probability of coasting before hitting the water. The density function will have a negative kurtosis to reflect this probability. The normal density function is given by

$$f(x) = \frac{1}{2\sqrt{\pi}} \exp\left(\frac{-(x-\mu)^2}{\sigma^2}\right) \tag{6}$$

with a typical mean (μ), a mode of μ, typical variance (σ^2) and no skew.

It is important to note that the normal distribution, due to its negative kurtosis, will have a larger standard deviation and a higher probability of mayday indicators in extreme ranges when compared to our log normal distribution. Thus the mayday indicator "hot spot" will be larger for a mechanical failure scenario and our search area will be expanded.

3 Modeling Crash Sites

3.1 Known Values

There is a thirty minute window between communications with pilots and ATC in which the aircraft can disappear (the disappearance interval). At each "check in" with ATC, a pilot must report their location, altitude, velocity, and intended path. Thus, from the last point of contact, we are aware of the aircraft's altitude and velocity. We can assume in the following thirty minutes that these will be held near constant as needless changes in velocity waste fuel.

3.2 Trajectory of Aircraft From Mayday Indicator

We will use the following equations of motion and their modifications as outlined by Wilson [12] to represent aerodynamic objects falling to Earth at an initial horizontal velocity.

$$\frac{du}{dt} = -g\frac{uV}{U^2}, \qquad \frac{dv}{dt} = g\left(1 - \frac{vV}{U^2}\right) \tag{7}$$

and

$$\frac{V^2}{r} = g\frac{u}{V}, \qquad \frac{dV}{dt} = g\left(\frac{v}{V} - \frac{V^2}{U^2}\right) \tag{8}$$

Where $U =$ terminal velocity, $g = 9.8m/s^2, u =$ horizontal component of velocity and $v =$ vertical component of velocity.

Through manipulation and various substitutions we can conclude

$$\frac{d^2y}{dx^2} = \frac{g}{u_0^2}e^{2gs/U^2} \tag{9}$$

for an arc s. To approximate this curve we can enclose it between two other curves, $x < s < x + y$. Thus we can model the trajectory as

$$\frac{d^2y}{dx^2} = \frac{g}{u_0^2}e^{2gx/U^2}, \text{ lies above the actual trajectory} \tag{10}$$

$$\frac{d^2y}{dx^2} = \frac{g}{u_0^2}e^{2gy/U^2}, \text{ lies above the actual trajectory} \tag{11}$$

$$\frac{d^2y}{dx^2} = \frac{g}{u_0^2}e^{2g(x+y)/U^2}, \text{ lies below the actual trajectory} \tag{12}$$

It is simple to integrate (10) and (11), the solutions are

$$y = \frac{U^4}{4gu_0^2}(e^{2gx/U^2} - 1) - \frac{U^2 x}{2u_0^2} \tag{13}$$

and

$$y = \frac{U^2}{g} \log\sec\frac{gx}{u_0 U} \tag{14}$$

Each of these will provide upper bounds to the actual trajectory, with (13) being most accurate for short trajectories and (14) being most applicable in the case that initial altitude is large.

We can solve (12) given the substitution of

$$z = x + y\frac{dz}{dx} = \frac{dy}{dx} + 1, \quad \frac{d^2z}{dx^2} = \frac{d^2y}{dx^2} \tag{15}$$

and so the solution of (12), and the lower bound to our trajectory, is

$$y + x = \frac{U^3}{g}\left[\log\sec\left(\frac{gx}{u_0 U}\sqrt{1 - \frac{u_0^2}{U^2}} + \sin^{-1}\frac{u_0}{U}\right) + \log\sqrt{1 - \frac{u_0^2}{U^2}}\right] \tag{16}$$

Depending upon the altitude of the flight, either (13) or (14) could be averaged with (15) to determine a probable crash point.

3.3 Determining Aircraft Crash Points

At cruising speed, the average long-distance commercial airplane flies at up to $900km/h$. In our disappearance interval, this leaves $450km$ as a maximum distance that the plane could have traveled. It is not feasible to search this entire area or to test the probability at each mayday indicator that the aircraft crashed.

Instead, we will test for ten probable mayday indicators based off of the PDF. Within one standard deviation of the mean of our PDF we will place seven probable mayday indicators (seventy percent of mayday indicators), within two deviations we will place nine mayday indicators (ninety percent) and within three deviations we will place ten mayday indicators (one hundred percent). This approximately parallels the $68 - 95 - 99.7$ rule associated to normal distributions. Mayday indicators will be placed such that the interval of the deviation(s) is spanned but areas of highest probabilities will receive the highest density of mayday indicators, see Figure 2.

We will then take each of these mayday indicators and apply our modified trajectory equations to determine the crash point. The probability assigned to each mayday indicator that the aircraft lost control at this point is now transfered to the crash site as the probability that the aircraft made contact with the ocean at this point.

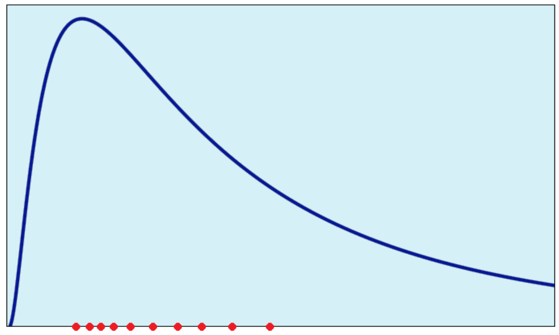

Figure 2: An example of how mayday indicators (red dots) might be assigned in a storm scenario

4 Modeling Aircraft Resting Location

After the aircraft has crashed and broken apart on impact, the fuselage and other debris will begin to move with the currents, wind and/or waves as they sink. We will once again employ modified forms of our trajectory equations from Section 3.2 to model the path of the fuselage as it sinks. It is important to note that the horizontal component of velocity is now dependent upon the outside factors such as ocean currents.

4.1 Important Values

To determine how the aircraft might be moved once it is in the ocean, we need to know some basic values.

4.1.1 Weight of Fuselage

The fuselage will immediately begin to lose pressurization after the crash as pressurized air is no longer being pumped into the cabin. Thus the weight of the fuselage [1],

$$W_{fuse} = (1.051 + .102xI_{fuse})V_{fuse} \tag{17}$$

will be determined by $I_{fuse} = \frac{I_p^2 + I_b^2}{2I_b}$ and its volume, V_{fuse}, where $I_p =$ pressure index and $I_b =$ bending index.

4.1.2 Inertia of Fuselage

The tendency for the fuselage not to move will be given by it's inertia [5]

$$J = \left(\sum_{i=1}^{n} m_i x_i^2 - x_{sc_k} \sum_{i=1}^{n} \right) \tag{18}$$

where

$$x_{sc_k} = \frac{\sum_{i=1}^{n} m_i x_i}{\sum_{i=1}^{n} m_i} = \text{center of gravity} \tag{19}$$

The fuselage will not be moved in any direction unless the inertia is first overcome.

4.1.3 Drag Force

The drag force will be an important component in determining how the aircraft moves in the water. Drag force is given by

$$F_d = c_d (1/2) \rho v^2 A \tag{20}$$

where F_d = drag force in Newtons, c_d is the drag coefficient equal to 0.045 [9], ρ = density of fluid, v = flow velocity and A is the characteristic frontal area of the body. It is important to note that our drag force will be a function of the density of the water which depends upon water temperature and salinity. Ocean water density can be averaged to $1027 kg/m^3$ at sea surface but the exact value should be calculated for the body of water the aircraft crashed into.

4.2 Possible Complications

4.2.1 Ocean Currents

Surface currents pose an interesting obstacle in the search of aircraft lost at sea. Once the aircraft has crashed, it can be carried away in many directions and at varying speeds, depending upon the currents present. Fortunately, currents are well documented and monitored with ample information on their strength and direction, as seen in Figure 3. To determine how far the fuselage is likely to travel on any given current, you simply need to know the velocity of the current, the weight of the fuselage and the drag in water (the latter two of which we have just shown how to calculate and the former of which is available from a variety of sources).

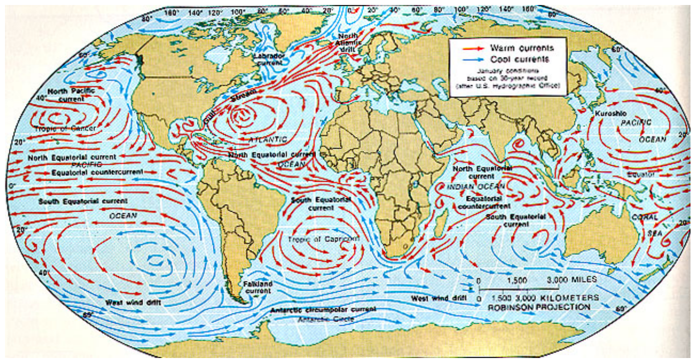

Figure 3: A visual representation of global ocean currents [8]

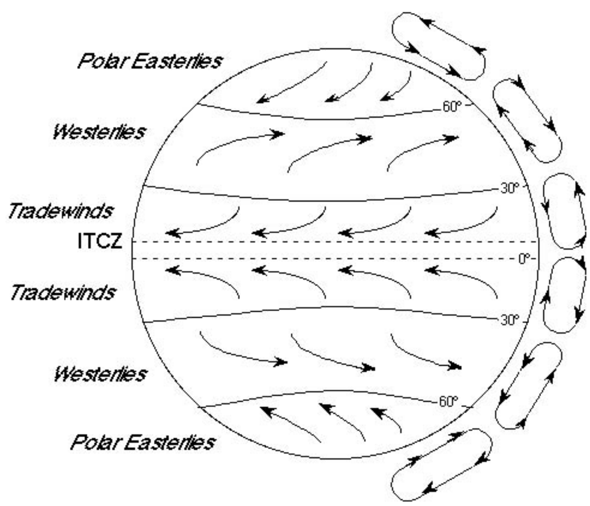

Figure 4: Global wind patterns [6]

4.2.2 Wind

The force of wind on the fuselage will be given by

$$F = APc_d \tag{21}$$

where A is the area of the object, $P = 0.00256v^2$ is the wind pressure, c_d is the drag coefficient and v is the velocity of the wind. If wind were to affect fuselage movement in water, this wind force would need to be greater than the inertia of the fuselage. Note however that the wind should have a minimal effect as the fuselage is aerodynamically designed and will not be moved like a sail boat. As such we will deal specifically with the currents when determining fuselage movement.

However, debris movement in water can be affected by wind patterns, this will play a large role in how to improve our model. It is important to understand that as wind is the driving force behind surface currents, the direction of the currents and wind are generally the same, as can be seen in a quick comparison of Figures 3 and 4.

4.2.3 Waves

Waves, while seemingly important, are not a major role in the movement of objects at sea. When a wave encounters an object on the water, it appears to move it forward. However, when the wave falls back, it takes the object with it back to its original location [11]. This can be seen when examining buoys floating in the waves. Due to this, the main role of waves in our model will be damaging the aircraft after it has crashed into the water, leading to faster sinking of the fuselage.

4.3 Forecasting the Sinking Pattern

There are two scenarios to consider when the fuselage is sinking: one in which the aircraft begins to sink immediately upon impact due to previously sustained damage and another in which the aircraft drifts with the currents before sinking. Note that in either scenario, the fuselage will sink as if in a "nose dive". This is due to the fact that as the fuselage fills with water, it will naturally tip in whichever direction is heavier and will sink in this manner. It is nearly impossible for the fuselage to be perfectly balanced when hitting the water and to stay in this equilibrium while filling with water and sinking, thus the case will be ignored.

4.3.1 Sinking Upon Impact

Given Wilson's modifications [12] of the equations of motion below we want to analyze the effects of sinking in a "nose dive" position on an aerodynamic object's horizontal and vertical trajectory.

$$\frac{du}{dt} = -g\frac{uV}{U^2}, \qquad \frac{dv}{dt} = g\left(1 - \frac{vV}{U^2}\right) \tag{22}$$

and

$$\frac{V^2}{r} = g\frac{u}{V}, \qquad \frac{dV}{dt} = g\left(\frac{v}{V} - \frac{V^2}{U^2}\right) \tag{23}$$

where once again $U =$ terminal velocity, $g = 9.8m/s^2, u =$ horizontal component of velocity, $v =$ vertical component of velocity and $r =$ radius of curvature of the path.

Rewriting the second portion of (23),

$$V\frac{dV}{dy} = g\left(1 - \frac{V^2}{U^2}\frac{V}{v}\right) \tag{24}$$

we can analyze the relation between the tangential and vertical velocities. It is easy to deduce that the increase in V with a vertical drop from y, will be less than the increase in v with the same vertical drop; the vertical velocity dominates the tangential velocity [12]. Hence, the direction of a solid body in downward free-fall will be dominated by the vertical velocity, minimizing the drag effects of the ocean current.

A further analysis considers the generalization to different mediums with varying resistance. We start by considering the arc of drift with respect to time. Noting that (22) may be integrated,

$$\frac{du}{dt} = \frac{du}{ds}\frac{ds}{dt} = V\frac{du}{ds}, \quad \frac{du}{ds} = -\frac{gu}{U^2}, \quad u = u_0 e^{-gs/U^2} \tag{25}$$

reveals that horizontal velocity decreases exponentially with respect to arc distance traveled and that

$$\frac{1}{r} = \frac{\frac{d^2y}{dx^2}}{[1+(\frac{dy}{dx})^2]^{3/2}} = \frac{\frac{d^2y}{dx^2}}{\frac{V^3}{u^3}} = g\left(\frac{u}{V^3}\right) \tag{26}$$

This calculation shows that

$$\frac{d^2y}{dx^2} = \frac{g}{u^2} \tag{27}$$

holds for all mediums of varying resistance [12].

Thus in this scenario, as the fuselage begins to sink immediately, it's location can be approximated almost directly below the crash point. We will define our search area as a circle of radius 3 nautical miles (NM) around this point.

4.3.2 Drifting Before Sinking

If the fuselage is intact enough to not immediately begin sinking and the force provided by the currents is strong enough to overcome the inertia, the fuselage will begin floating in the direction of the currents. However, a fuselage can only float for a certain amount of time. Aircraft are not built air- or watertight, instead they are sealed to keep high pressure air inside the cabin with pressurized air being constantly pumped in. In the event of a crash, the cabin becomes de-pressurized. The seals are then forced to work to prevent higher pressure water from entering, a task they were not designed to do and will ultimately fail at. Even in a fuselage undamaged by the crash, water will immediately begin to enter the cabin, affecting the buoyancy of the fuselage.

Once the buoyant force is less than the force of gravity:

$$F_b = g\rho V < gm \tag{28}$$

for ρ = water density, V = volume, g = gravity and m = mass, the fuselage will begin to sink. Knowing this, we have calculated that it is highly unlikely that an object will drift more than one nautical mile from its original crash site before beginning to sink. It is simply too improbable that the fuselage is in good enough condition after the crash to maintain buoyancy longer than that. Thus the search radius outlined above will encompass any possibility of drifting and will work in either scenario.

Note we have ignored the possibility that a storm could be present and altering the drifting path. While a powerful storm could carry the fuselage farther than any currents might be able to in the same amount of time, it will also damage the fuselage more dramatically. This will lead to a more rapid loss of buoyancy and, ultimately, the fuselage sinking even faster than if the storm were absent.

5 Locating Aircraft in Search Areas

5.1 Application of Bayesian Search Theory

To optimize our search for the missing aircraft we will apply Bayesian search methods based on our probabilities determined in the above sections.

Let S be the set of all of our search areas, bounded and disjoint. We have assigned a probability, p_i, of the aircraft being in any S_i using our probability density function. For each S_i there exists another probability, q_i, of finding the aircraft. This q_i is dependent upon many factors, such as the depth of the water and the effectiveness of the chosen search vehicle. Bayesian search theory states that if we check any of our search areas and do not find the aircraft, the probability, p_i', of it being in S_i is now

$$p_i' = \frac{p_i(1-q_i)}{(1-p_i)+p_i(1-q_i)} = p_i \frac{1-q_i}{1-p_i q_i} \tag{29}$$

and the updated probability, p_k', of the aircraft being in some other S_k is

$$p_k' = p_k \frac{1}{1-p_i q_i} \tag{30}$$

Note that $0 < p_i' < p_i < 1$ and $p_k < p_k' < 1$. Hence the probability of the aircraft being where we have already searched decreases (but does not vanish) and the probability of the aircraft being where we haven't checked increases. Our probabilities will be constantly updated, giving us the highest chance of locating the aircraft in the minimal amount of time.

5.2 Recommended Search Vehicles

In any downed aircraft scenario, we recommend that unmanned aerial vehicles (UAV) are immediately deployed to scout for potential survivor rafts and/or obvious debris. UAVs are fast and able to scour large areas quickly without risk of human injury. The next course of action we recommend depends upon if survivors are found or not.

5.2.1 If Survivors are Found

In the case that survivors are present, we recommend the tactic outlined by Furukawa et al in their paper on Bayesian search for multiple targets [4]. In short, this paper tests the effectiveness of a combined effort between UAVs and helicopters with rescue workers. The UAVs fly ahead of the helicopters to locate the survivors and fly in circles around the survivors, lighting the area and alerting the helicopters to their location. The helicopters and rescue workers can then begin collecting the survivors, bringing them back to a safe location. The paper shows the combined efforts of UAVs and helicopter search and rescue teams to be quite effective in the location and rescue of survivors. Once each passenger has been found and brought to safety, we recommend the search proceed as if no survivors are found.

5.2.2 If No Survivors are Found

In the case that no survivors are found, the main priority becomes the location of the fuselage so that bodies of passengers and the black box can be collected. In this scenario, we recommend that underwater vehicles are deployed to search at least two areas of highest probability. This allows for the timely search of multiple areas without requiring the simultaneous search of all areas, which could become costly and may not be feasible for the search team. If the fuselage is found, searching is halted as bodies and important equipment are brought to surface. Searching would continue with last calculated probabilities if a passenger or some item of value is determined missing.

6 Testing the Model

Our model relies on specific information (wind speed, flight location, storm locations, etc) available at the time of the crash and within a limited time frame after. As this data was not recorded, or at least is not available to the public, for well documented cases (such as Malaysian Airlines Flight MH370 and Air France Flight 447) we unfortunately cannot apply our model to them.

With this in mind we attempted to simulate an aircraft crash so that our model could be tested. However, due to a lack of tools and advanced technology at our disposal, our simulations were reduced to trivial problems that did not challenge our model. The simulations were too predictable and did not accurately represent the variability in an actual aircraft crash. We could not in good faith claim that these simulations truly tested our model.

Having said this, we firmly believe in the approach our model employs. We are confident that, given the right tools and technology, a simulation could be constructed that mirrors realistic circumstances that could then test our model for accuracy. In this scenario, we recommend the following course of action:

- Construct a probability density function based on the chance of mechanical error or storm interference as outlined in Section 2.
- Using this PDF, determine potential mayday indicators.
- From each of these determined crash locations, apply our modified trajectory equations from Section 3.2 to determine the crash points.
- Define a search area around each of these points of radius $3NM$.

We are confident that, when tested under realistic circumstances, our model will prove to accurately predict resting location of the aircraft.

7 Improving the Model

No model will be able to accurately describe the exact location of an aircraft's crash in the ocean, there are too many moving variables. However, we have outlined a few ways in which our model could be improved and how technology could be improved for a better application of our model.

7.1 Recursive Tracking

Often, an aircraft crashes into the sea and, for one reason or another, cannot be located for a long period of time. Perhaps the most famous cases would be of Malaysian Airlines Flight MH370 and Air France Flight 447. While Flight MH370 is still missing, Flight 447 was found two years after the crash by a team who developed and employed a recursive tracking method [2]. In short, the movement of bodies and debris were traced backwards in time. This was done through the use of a three-dimensional Lagrangian tracking program

$$P_n(\bar{x}_{t-\Delta t}, t - \Delta t) = -\int_{t}^{t-\Delta t} (\bar{v} + \alpha\bar{w})dt + P_n(\bar{x}_t, t) \qquad (31)$$

where $P_n(\bar{x}_{t-\Delta t}, t-\Delta t)$ and $P_n(\bar{x}_t, t)$ are the locations of the nth object at time $t - \Delta t$ and t, $\bar{v}$ is the velocity vector, $\bar{w}$ is the wind velocity vector and α is the wind drag factor.

The application of this technique to our model would help with creating even more accurate representations of probabilities of crash sites, leading to the even faster location of the aircraft. This method would also allow our model to be used in cases it was not designed for, such as in a plane hijacking or intentional crash.

7.2 Application of Global Tracking Systems

It will come as no surprise that the accuracy of our model is dependent upon accurate tracking systems. Radar, while common, is no longer the best method for tracking aircraft, especially over the ocean. Instead, we recommend the application of improved tracking systems. A promising system, the NextGen satellite system, tracks aircraft using Automatic Dependent Surveillance-Broadcast (ADS-B). The NextGen system has already been employed by several aviation administrations, with the Federal Aviation Administration (FAA) deploying it nationwide in 2013 [3]. The NextGen system was combined with other tracking systems by Liu et al. in an attempt to improve aircraft tracking through data fusion. [7]. The multiple data sources and algorithms run created a highly accurate system with a low sensor fault. Applications of either of these two tracking systems, or another promising tracking system, would not only increase the accuracy of our model but reduce the probability of crash in the first place.

8 Conclusion

Locating an ocean downed aircraft to a precise point is unrealistic. Therefore, the task becomes one of maximizing search efficiency. We propose a model based on the assumption that pilots make contact within specified time intervals. Our model then calculates a maximum search area given data retrieved from the plane at point of last contact: velocity, trajectory, and weather conditions. We then use scenario analysis to model the distributions of the possible crash sites finding that different scenarios would suggest different probability distributions. Our model uses Bayesian search methods to maximize search efficiency after having established the hypotheses and assigning a distribution function to the search area. Lack of sufficient data has made testing the model challenging and final results are inconclusive. However, we would like to stress our confidence in the approach that the model employs and emphasize our belief that further testing will reveal positive results.

9 Executive Summary

As is understood, the priority in any downed aircraft scenario is salvaging human life. Due to the nature of aircraft crashes, often this is not a possibility and the next priority becomes locating the bodies of the passengers. With this in mind, after an aircraft is presumed downed in the ocean, the most critical success factor is time. In view of recent cases of ocean downed aircraft, it is understood in our industry that there is a dire need for more precise and efficient search methods. Our team has developed a method that maximizes the efficiency of search and rescue efforts.

Our method is based on universal principles of motion which have been tested and proven over centuries. Our research and development team has developed a model based on **search efficiency** that will greatly narrow ocean search areas. We have outlined an implementation strategy that can be followed and used by anyone in charge of search and rescue. In this report we outline the methods we will undertake to maximize search efficiency in locating ocean downed aircraft. We outline this method as follows:

- **Identifying a maximum crash site area:** International flight protocol mandates that pilots make periodic contact. Utilizing universal laws of motion, our model forecasts a crash site located within a bounded region. This increased precision increases search productivity by eliminating areas that have no probability of containing the aircraft.
- **Prevailing ocean patterns:** The most difficult task in forecasting the location of ocean downed aircraft is the dynamic nature of ocean currents. After the aircraft makes contact with the ocean, we have to consider the effects of the currents to determine where the craft is likely to be located. Although we know that this can't be achieved with perfect precision, our model determines a maximum drift distance from the crash site. This allows us to apply statistical search methods and increasing productivity of search and rescue teams.
- **Implementation of the Model:** As with any new method or process, successful implementation is critical to the success of the endeavor. As many new great ideas never succeed, implementation can be viewed as one of the most challenging components of any new idea, much more so with complicated mathematical models. To this end, our model is designed with simplicity and adaptability in mind. We make no attempts to reinvent the wheel. Instead, we use proven methods that have been slightly modified to fit the scenario of an ocean downed aircraft. Our model works with known variables that can be input by anyone

trained in search and rescue. The model will be implemented in search and rescue training procedures and will be used in simulations effective immediately.

As recent events highlight the need for improvement in open ocean search efficiency, we have developed this model and methods for our clients. We are confident that with the application of the techniques heretofore outlined, we will dramatically increase search efficiency for ocean downed aircraft. Moreover, the overall task of salvaging human life in such events is the ultimate goal; we are also looking at ways of harnessing the wonders of technology to assist in our efforts. The recursive techniques used in the case of Air France Flight 447 and continuous satellite tracking are innovations that we will also utilize in our search and rescue efforts. Thank you for your trust, patience, and support in our efforts to not only increase search efficiency and productivity for ocean downed aircraft, but more importantly, in salvaging human life.

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