Summary

Given the tasks to evaluate the performance of a driving rule that people should always stay on the right most lane unless they are passing. It's always hard to evaluate a rule. The direction that we are approaching this problem is to first construct the traffic flow model then implement the rule on the traffic flow to examine it's performance. Intuitively, dynamics of traffic flow is, on some level, similar to the dynamics of fluid. However, our approach in a discretized model based on Cellular Automaton, provides an organized algothrithmic steps to construct our traffic flow. In this traffic flow model, cars are either occupying a cell or not. It's also easy to track cars't bevaior on road in terms of their exsistance in a cell. Not surprisingly, we found that flow of traffic is proportional to the density of traffic until the critical point of density. After critical point of density, the flow of traffic is in jamed state. With the driving rule enforced as a preference parameter on the the behavior of changing lane, we can see that flow of traffic is limited under the driving rule.

The Keep-Right-Except-To-Pass Rule

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Contents

1 Problem

The task given is to evaluate the performance of a driving rule: drivers are required to drive in the most right lane unless they are passing another car, in which case, they can move one lane to the left, pass, and then return to their formertravel lane. In order to evaluate of this driving rule, we first need to model the traffic flow on the road. Traffic stream is a complex and nonlinear system and is multidimensional[3]. Intuitively, one would start construct continuous model for traffic flow, becuase it's dynamics somewhat resemble the fluid dynamics[1]. However, we will introduce a different approod by discretized the traffic flow. With the model of traffic flow, we'll be able to vary parameters that enforces the driving rule in dynamics of traffice flow, and evaluate driving rule from there.

2 Model

The discretized traffic model is based off celluar automaton model. Nagel and Schrekenber(1992) were the first people who conducted research on traffic simulation using Automaton model. To analysize traffic flow, there are three main characteristics of traffic that we need to define, and they are speed, flow, and density[2].

Speed (V) is defined as distance travelled in a unit time. Density (K) is defined as the number of cars per unit area. Flow (F) is defind as the number of cars per unit time. Intuitively, we can see that there is a inverse realtionship between Flow and Density. The relation is usually defined as $Q = V * K$.

In Cellular Automaton model, road consists of a grid of cells and cars are located in some cells on the road. Cars move from one cell to another at each time step. Each cell on the road can be either occupied or empty. The traffic flows as at each time step each vehicle will move some number of cells. Velocity in this model is defined as an interger from $[0, 5]$, which $v_{max} = 5$ corresponds to a real velocity of 135km/h, or 84 mph.

There are four steps movement completed at each unit time:

Step 1. Acceleration: All the cars that have't reached the maximum v_{max} will accelerate by some rate.

$$v_i \to min(v_i + a, v_{max}) \tag{1}$$

Step 2. Deceleration: Cars reduce speed if the front gap is not enough for current speed. The speed will be reduced to $gap_i - 1$.

$$v_i \to min(v_i, gap_i - 1) \tag{2}$$

where

$$gap_i = x_i - x_{(i-1)} \tag{3}$$

Step3. Randomization: Driver will randomly decrease their current driving speed by one unit. With the probability of this randomization is p, we define:

$$v_i \to max(v_i - 1, 0) \tag{4}$$

Step 4. Move: after previous 3 steps, the new position of the car can be determined by the current speed and it's current position.

$$x_i \rightarrow x_i + v_i \tag{5}$$

We first simulate the one-lane traffic based on the basic four rules.

To get a model of a traffic flow that is more realistic. We then model double-lane highway traffic based on the rules of one-lane highway, with additional rule for lane changing. A car is allowed to change lane if there is no car in the taget lane at the time it will make movement. When a car is changing the lane, it's only making horizontal movement at that time step. More specific rules of double-lane highway traffic are stated below.

1. Acceleration: Cars that haven't reached to maximum speed will accelerate by some rate.

$$v_i \rightarrow min(v_i + a, v_{max}) \tag{6}$$

2.Deceleration: Cars reduce speed if the front gap is not enough for current speed. The speed will be reduced to $gap_i - 1$.

$$v_i \rightarrow min(v_i, gap_i - 1) \tag{7}$$

where

$$gap_i = x_i - x_{(i-1)} \tag{8}$$

3. Randomization: Driver will randomly decrease their current driving speed by one unit. With the probability of this randomization is p, we define:

$$v_i \rightarrow max(v_i - 1, 0) \tag{9}$$

4. Swith lane: Cars will change lane under certain criteria.
(a). The gap between i^{th} car of the car ahead of it is smaller than the i^{th} car's current speed.
(b). The distance of the car ahead in the target lane is greater than the distance of car ahead in the current lane.
(c).There is a parallel empty cell in the target lane.
(d).The distance ahead of the following car in the target lane is greater than the current speed that following car.

Usually, criteria (a) and (b) are known as incentive critera and is defined as follow:

$$gap_i < v_i \tag{10}$$

$$gap_{pred} < gap_i \tag{11}$$

Critera (c) and (d) are known as safety rules that will ensure the movement of lane changing will not cause accident. The safety criteria are defined as:

$$gap_{succ} < gap_{safe} \tag{12}$$

5. Move: the new position of the car can be determined by the current speed, it's current position and movement of changing lanes.

$$x_i \rightarrow x_i + v_i \tag{13}$$

3 Mathematical Derivation of Cellular Automaton Traffic Model

We have shown algorithms of the discretized traffic model, but does it mathematically make sense too?

Intuitively, before K_c, critical value of density, flow increases as density increases since the overall traffic flow is at free flow state. After K_c, the flow decreases as density increases since congestion occur which will eventually lead to traffic jam as density increases. When the traffic flow is close to D_c, the cars are more likely to travel at V_{max}. We have stated before that the probability of a car decreasing it's speed is p. Therefore, the average speed of free flow is:

$$v_f = (1-p)v_{max} + p(v_{max} - 1) \tag{14}$$

Then, flow of traffic can be expressed as:

$$Q = (v_{max} - p) * K \tag{15}$$

If we divide the density K into two state: free flow state K_f and traffic jam state K_j. Further detailed derivation can be found in Barlovic, Santen and Schadshneider. Eventually, we will get, Critical density,

$$K = \frac{(1-p)}{(V_{max} + 1 - 2p)} \tag{16}$$

P, again, is the probability that a car will slow down.

Max Speed (p=0)	Critical density of the Block(Simulation)	Critical density of the Block (Formula)
1	0.675	0.68
2	0.476	0.5
3	0.324	0.3
4	0.301	0.3
5	0.145	0.15

Figure 1: Critical density of the Block

The data in the table clearly showed us the how different value max speed would influence the critical value of density. When the speed is large, the critical density value decreases. This implies that if cars travels at higher speed, it's easy for traffic flow to become congested. When the driving rule is enforced on the flow, the system will become even more easily to get into conjestion.

References

[1] Saifallah Benjaafar, Kevin Dooley, etc.*Cellular Automata For Traffic Flow Modeling*, Center for Transportation Studies, Minnesota(1997).

[2] Ding Ding,
Modeling and Simulation of Highway Traffic Using a Cellular Automaton Approach, URL: `http://uu.diva-portal.org/smash/get/diva2:483914/FULLTEXT01.pdf`.

[3] Peter Wagner, Kai Nagel, etc. *Realistic Multi-lane Traffic Rules For Cellular Automata*, PHYSICA A (1996); vol.234, pages 96–2586