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Radio Experts Perplexed Engineering A Thoroughly Efficient Repeater System

Control #10500

Abstract

This paper examines the challenges of amateur radio network planning by exploring the simultaneous usability of repeaters by a large user population. We attempt to minimize the number of repeaters necessary to service an area while providing user satisfaction by efficiently allocating the available number of frequencies while preventing co-channel interference. We focus on examining scenarios of 1,000 and 10,000 concurrent users. Our model utilizes probability, geometric number theory, and optimization in order to approximate the necessary number of repeaters. Our results show that our network will require a minimum of 4,158 repeaters for the case of 1,000 simultaneous users and 24,651 repeaters for the case of 10,000 users, while satisfying the constraints of servicing the entire region. Through our model, we discovered that it is completely impossible to entirely eliminate co-channel interference throughout the region. Thus, we explored the most efficient method for allocating frequencies.

1 Introduction and Statement of Problem

1.1 Introduction

Radio enthusiasts around the world use amateur radio frequency bands for personal communication and exchange of information. Due to local and federal regulations restricting the allowed power usage of amateur radio transmitters and limited equipment size, **the broadcast distance range of any single ham radio is limited.** This range is further confined by topographical obstruction and atmospheric conditions. **To extend transmission range, operators must rely on repeaters to amplify broadcast signals.**

Providing further complication, amateur frequency usage is restricted by the FCC to a small band of frequencies between 145-148 MHz. In order to avoid signal interference, operators wishing to broadcast simultaneously while in signal range of one another must all transmit on different frequencies. However, at times of high transmission in areas with more operators than usable frequencies, **the available number of frequencies may not satisfy simultaneous operation. The use of Private Line (PL) tones can alleviate this problem.** In a radio network utilizing repeaters, a PL tone assigned to each repeater can act as a restraining factor in the transmission of a signal. A signal received by the repeater must not only be tuned to the repeater's frequency, but must also include a sub-audible tone on the frequency of that repeater's PL tone. If either the frequency or the PL tone reaching the repeater do not match those programmed into the repeater, the repeater does not transmit the signal. This allows two repeaters tuned to the same frequency to be placed near one another without causing interference as long as they have different PL tones.

1.2 Statement of Problem

When constructing a network of PL tone repeaters, one must make the following considerations:

- Repeaters within signal range of each other cannot transmit on the same frequency and PL tone without causing interference. Repeater range is roughly 25 miles.
- Areas of higher network usage require more repeaters than areas of less network usage in order to provide coverage for all operators and minimize interference.
- Transmitter broadcast range is limited.

In this paper, we address these problems and outline a mathematical method for providing complete repeater coverage of an area of roughly 5,000 square miles for 1,000, and later 10,000, simultaneous users while minimizing the necessary number of repeaters.

2 Plan of Attack:

Our intention is to develop a geographic model that determines the minimum number of repeaters necessary in a circular area to accommodate a given usage load. In order to accomplish this, our model must consider the location of radio operators within the area bounds, and repeater locations required in order to provide complete area coverage for all radio operators. We will move toward this objective in the following manner:

1. **Establish Assumptions and Terms:** In order to create a workable model, we require the construction of several assumptions about available equipment and the usability of this equipment in our given area. Outlining these assumptions provides a better understanding of our method.
2. **Present Our Model:** Given our assumptions, we simulate a simplified version of the problem.
3. **Compare Our Model to Models of Similar Problems:** This will indicate the limitations of our model and suggest other possible avenues for adjustment under different constraints.

3 Terms and Assumptions

3.1 Terms Defined

The terms below will be defined as follows for the remainder of the paper:

1. **Signal:** A signal refers to a single electromagnetic wave and hence a single frequency (rather than a superposition of waves or frequencies) sent out from any piece of equipment.

2. **Repeater:** A repeater is an antenna that simultaneously receives, amplifies, and transmits a signal.

3. **Transmitter:** A transmitter refers to an amateur radio broadcasting without the use of a repeater.

4. **Signal Range:** The signal range of a transmitter (or of a repeater) refers to the farthest radial distance a signal can travel from this transmitter while still remaining distinguishable from noise according to the sensitivity of the receiving repeater (or transmitter).

5. **PL Tone:** A PL (Private Line) tone is a sub-audible tone in the range of frequencies between 67-254 Hz that is superposed with a signal and transmitted simultaneously. There are 54 PL tones.

6. **F-T Pair:** An F-T pair refers to the superposition and simultaneous transmission of a signal of specified frequency and a PL tone.

7. **Interference (Co-Channel Interference:** Interference occurs when two signals of the same frequency interact with one another. In the case of this model, we use this word to refer to any situation in which the interaction between two signals sent out by two repeaters (or two transmitters, or a transmitter and a repeater) causes the destruction of communication between a transmitter and a repeater. For example, when two repeaters within each others' signal range send signals on the same frequency, an operator also in this range trying to receive a signal on this frequency cannot distinguish between the two signals, and thus cannot obtain any information (such as voice or Morse code) from either signal.

8. **User:** A user is defined as a operater of an amateur radio attempting to broadcast through a transmitter. *Users are only broadcasters; they do not receive any signals.*

9. **User Satisfaction:** A measure of user satisfaction is minimizing the area where a user is unable to broadcast. This requires that there is a frequency available for their use, and that they can transmit as far as possible.

10. **Reuse Distance:** This is the minimum distance needed between two repeaters to prevent co-channel interference.

3.2 Assumptions

In order to define our problem in workable terms, we employ the following assumptions about available equipment, the functionality of this equipment, and the nature of the area in which our network is to be constructed:

1. **Our network is located in an urban area.** This assumption is based on the given conditions of amateur radio operator populations of 1,000 and 10,000 in an area of roughly 5,000 miles, and on published data estimating the number of ham radio operators in U.S. cities [2].

2. **Transmitters have an average signal range of roughly 5 miles.** Radio operators can obtain licenses that allow them to broadcast on equipment with varying levels of power. A different license is required to broadcast with the highest level of equipment power than to broadcast with a lower level of power. The highest legal level of power an operator may use is 1500 Watts [4]. However, operators are legally restricted to use the minimum transmitter power necessary to carry out the desired communications [4]. Since typical transmitter power usages range from 5 W to 1500 W, and considering minimum transmission laws, we assume henceforth that transmitters typically operate with average power usage of 10 W. Thus, using the equation:

$$L = 40 \log d_m - 20 \log h_T h_R$$

   where L is the loss of power of a signal traveling in an urban area, d is the distance traveled by the signal, h_t is the height above ground of the transmitter, and h_r is the height above ground of the repeater, we estimate that a transmitter signal attenuation is large at a distance of 5 miles from the source [1]. Thus, **we assume that a transmitter must be within 5 miles of a repeater in order for the repeater to receive and further transmit a signal from this transmitter.**

3. **Repeaters amplify signals up to a distance of 25 miles.** Using the following equation, which describes the attenuation of a signal traveling through free space:

$$L = 32 + 20 \log f_{MHz} + 20 \log d_{KM}$$

   where L is the loss of power, f_{MHz} is the frequency and d_{KM} is the distance [1]. We estimate that a repeater's signal can be received by a transmitter located a distance of 25 miles from the repeater. Note that we approximate that the repeater signal travels through free space, as repeaters are typically placed at much higher elevations than transmitters, thus eliminating many of the barriers encountered at lower elevations.

4. **Only the interaction of two signals of the same F-T pair causes interference.** Interacting signals of the same frequency but different PL tone, or of different frequency and the same PL tone do not interfere.

5. **Repeaters are tuned to amplify signals of only one F-T pair.** Any transmitter wishing to amplify a signal using a repeater must broadcast using both the frequency and PL tone of the repeater. If either the PL tone or the frequency sent by the transmitter do not match that of the repeater, the repeater will not amplify the signal. This, along with Assumption 3, implies that after 50 miles a F-T pair can be reused but within 50 miles the same F-T pair will cause interference.

6. **Only one transmitter can utilize a repeater at an instant in time without causing interference.**

7. **Two repeaters using the same F-T pair cannot be placed within each other's signal range without causing interference.** Thus, any two repeaters using the same F-T pair must be positioned no less than 50 miles from each other.

8. **A repeater receives and transmits a signal simultaneously without interference between its received and broadcasted signal**. The repeater accomplishes this by increasing (or decreasing, depending on the received frequency) the frequency of each received signal by 600 Hz.

9. **Amateur radio transmitters can transmit signals only on the following frequencies: 145.01-145.39; 146.01-146.37; 147.6-147.99.** Channels must be spaced 0.02 MHz apart from each other [5]. This allows us to have available 56 available channels. **With the PL tones, the total available F-T pairs are 3024.** They cannot make use of all frequencies legally allowed for amateur use, i.e. 145-148 MHz, due to the 600 Hz change in amplified frequency used by repeaters. Repeaters accept the frequencies listed above and add 600Hz to 145.01-145.39 MHz and 146.01-146.37 MHz while subtracting 600Hz from 147.6-147.99 MHz [5]. The gaps in transmitter frequency usage prevents a transmitter from sending a signal on the same frequency as an amplified repeater signal, thus limiting interference.

4 Building the Model

4.1 The Strategy

The strategy is to structure the model around an optimization problem:

min The number of repeaters
s.t. User Satisfaction
Controlling Interference from within signal zone
Allocation of F-T pairs effectively

Instead of tackling the optimization problem directly, we focus at first on statisfying the constraints and in the process determine the limiting factors that control the number of repeaters in a region.

4.2 Distributing the User Population

As described before, users transmit their signals through repeaters. However because of interference, if a user wants to ensure their signal is clearly received, they must use a separate F-T pair than other users. Otherwise, if two users use the same F-T pair (within signal range), the messages will merge and become incomprehensible. Therefore, if users are to simultaneously use the communication network, each user within the signal range must have a distinct F-T pair. Since repeaters can only amplify one F-T pair, each of these users within the signal range must have their own repeater. Hence, it is imperative to understand the location of users.

To satisfy the constraint of an available repeater within a transmission range for all users, we must understand the dispersion of the population across the 5000 sq. miles. This allows us to provide a minimum bound on the number of repeaters needed to service a given area, as we need at least that many repeaters to handle the expected number of users. The upper bound will be determined through probability distributions.

4.2.1 A Simple Model

For a simple model, we assume the users are uniformly distributed across the region. In particular we will focus on the 1,000 user case and amend the model as needed for the 10,000 case.

We would like to determine with 99% certainty the number of users within a particular radius, namely the range of a transmitter 5 miles. To achieve this we will need to define variables:

1. $x_1, x_2, \ldots, x_{1000}$ represent the x_i user. Because the users are uniformly distributed they are iid.
2. $N_i =$ the number of $\{x_{j_{j \neq i}} \in B_5(x_i)\}$
   The number of x_j's that lie in a ball centered at x_i.
3. N is the max of N_i

The goal is to find a t such that $\mathbb{P}(N > t) < .01$ where t is the maximum number of users within a region with 99% confidence.

Union Bound
The *Union Bound* in probability allows us to approximate the $\mathbb{P}(N > t)$. It states that:

$$\mathbb{P}\left(\bigcup_i A_i\right) \leq \sum_i \mathbb{P}(A_i)$$

for a countable set of events $A_1, A_2, \ldots$. We are able to apply such a theorem because

$$\begin{aligned}\mathbb{P}(N > t) &= \mathbb{P}(\exists\ i \text{ s.t. } N_i > t)\\ &\leq \sum_{i=1}^{n} \mathbb{P}(N_i > t)\\ &= n\mathbb{P}(N_1 > t) \qquad \text{due to independence.}\end{aligned}$$

Focusing attention onto $\mathbb{P}(N_1 > t)$. We note this is equivalent to saying

$$\mathbb{P}(x_{s1}, x_{s2}, \ldots, \in B_r(x_1) \text{ and the others do not}))$$

where $S \subseteq n$. Therefore this defines a binomial distribution with $E[X] = np$. In our case because of the uniform distribution, **we know that the** $\mathbb{P}(x_i \in B_5(x_i)) = \frac{5^2}{1600}$ **or more generally** $\frac{r^2\pi}{1600\pi}$ **for a ball of radius** r**.** We now see that n is significantly large and p is significantly small, allowing the approximate value of $\mathbb{P}(N_1 > t)$ to be computed by a Poisson with $\lambda = \frac{nr^2\pi}{1600\pi}$

Poisson Distribution Approximation

In terms of a Poisson,

$$\mathbb{P}(N_1 > t) = \sum_{k=t+1}^{1000} \frac{e^{-\lambda}\lambda^k}{k!}$$

Approximation of a upper bound on a Poisson is given by:
Chernoff Bound
For any $\delta > 0$

$$\mathbb{P}(X > (1+\delta)\mu) < e^{(-\mu\delta^2/4)}$$

where X is the sum of independent Poisson trials and $\mu = np$. [3]

Substituting $(1+\delta)\mu) = t$, we achieve the upper bound as

$$\mathbb{P}(N > t) < e^{(-\mu(\frac{t}{\mu}-1)^2/4)}$$

We are now able to solve for an upper bound:

$$n\mathbb{P}(N > t) < n * e^{(-\mu(\frac{t}{\mu}-1)^2/4)} < .01$$

allowing us to explicitely solve for t.

How Good is the Approximation

We see that

$$\mathbb{P}(N_1 > t) < \mathbb{P}(N > t) < n\mathbb{P}(N_1 > t)$$

given that n is a significantly large number and the distribution is Poisson. The Poisson distribution implies that $\mathbb{P}(N_1 > t)$ is small and therefore the n will not significantly affect the value. Thus, we can conclude that this is a fairly reasonable approximation to for $\mathbb{P}(N > t)$.

Computing the Values
We would like to find the maximum number of people within one transmitter range, or 5 miles with 99% probability.

1. ***Users = 1,000***
   This implies that $n = 1000$ **and solving for** $t = 42.44$
2. ***Users = 10,000***
   This implies that $n = 10,000$ **and solving for** $t = 249.173$

We are 99% sure that the maximum number of users within 5 miles of each other is 42 and 249 for 1000 and 10,000 users respectively.

4.3 Covering the User Population

In order to address the problem of providing repeater coverage for a given number of simultaneous users, we conclude the following based on our assumptions established in Section 3.2:

- Relying on Assumption 6: *Only one transmitter can utilize a repeater at an instant in time without causing interference*, we infer that **complete, simultaneous repeater coverage requires that each transmitter be assigned at least one repeater**. This is the only way to ensure that each user is able to transmit simultaneously without the destruction of signal due to interference.
- Assumptions 5: *Repeaters are tuned to amplify signals of only one F-T pair*, and 6 also lead us to conclude that the *first* signal of the correct F-T pair to reach a repeater is the *only* signal that repeater will transmit at a given moment in time. Also relying on Assumption 1: *Our network is located in an urban area with approximately* uniform *signal attenuation*, we then determine that **the repeater assigned to a transmitter will be the closest repeater to that transmitter. The transmitter will thus broadcast using the F-T pair of its assigned repeater.**

The problem of simultaneous coverage thus becomes one of pairing repeaters with transmitters, and placing repeaters of the same F-T pair at distances of at least 50 miles from each other (Assumption 7), **while accounting for the probability distribution of transmitters.**

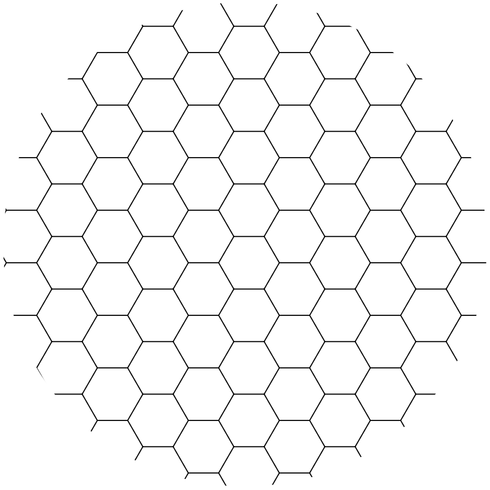
To address these problems, we model the area of our network as a circular plane with a lattice covering of regular hexagons (cells). The circular area in Figure 1 represents the 40 mile radius circular area of our network. Based on Assumption 2: *Transmitters have an average signal radius of 5 miles,* we assign the radius of each hexagon within this circle to represent a distance of 5 miles.

Utilizing the theorem from geometric number theory, proved by L. J. Toth, that states:

> *The densest packing of equal discs in the plane is obtained by arranging them in a hexagonal pattern with each disc touching six others* [8]

we claim that our network thus mapped represents the most efficient method for covering an area with densely packed circles while minimizing overlap.

Figure 1



Relying on the probability density of our user population, we assume that for the case of 1,000 users, the *largest* number of transmitters within a 5 mile radius of each other is 42, and for the case of 10,000 users, 249, both with 99% certainty. Thus, at any given time in the case of 1,000 users, each hexagon contains a maximum of 42 transmitters, and in the case of 10,000 users, a maximum of 249 transmitters.

The total number of hexagons within our network area is 99, thus implying that for the case of 1,000 users, the maximum required number of repeaters is 4,158. For 10,000 users, the maximum number of necessary repeaters is 24,651.

4.4 Distribution

We now have an estimate on how many repeaters are necessary to guarantee that every user has an available repeater to transmit their signal. ?Notice that the needed amount of repeaters is much higher than the amount of different F-T pairs. Thus its impossible to completely prevent co-channel interference as well as have an available repeater for everyone. We thus attempt to create a distribution that maximizes User Satisfaction.

4.5 Theoretical

This problem is a simple case of a more general problem of radio network allocations. This simple case is called the *Fixed Spectrum Assignment*, as in the amount of channels per cell is fixed and the channels themselves are fixed. This is exactly our scenario since repeater's location and input F-T is fixed.

We exemplify our method through the following example.

You have 5 different cells with a uniform population distribution. Let there be 5 users per cell using a repeater at any given time. If you can't have any frequency repeated in an adjacent cell, how would you distribute the repeaters? You need 5 different frequencies per cell and each of those frequencies can not be repeated in any adjacent cell due to co-channel interference. We encode these contraints in a $N \times N$ constaint matrix I, where N is the number of cells that need to be covered, such that each entry is either

$$I(i,j) = \begin{cases} 1 & \text{if ith cell adjacent to j;} \\ 0 & \text{if not.} \end{cases}$$

Thus, for our example,I is a 5×5 matrix,

$$I = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

We also define a $N \times 1$ matrix T where each row represents a cell and the entry represents the allowed amount of colors in the cell, thus T in this case is simply

$$I = \begin{bmatrix} 5 \\ 5 \\ 5 \\ 5 \\ 5 \end{bmatrix}$$

Now define another binary matrix $A = N\times$ n matrix where n is the number of available frequencies. We call A the distribution matrix. If the entry $A(i,j) = 1$, then we say that the *ith* cell as the *jth* frequency.

We attempt to minimize the amount of co-channel interference while guaranteeing a repeater for every user by setting the constraint

$$\sum_{i=1}^{n} A(i,j) \; = T$$

and minimizing the following operation on the matrix A.
If $a_1 = \mathrm{A}(i_1, j_1) = 1$ and $a_2 = \mathrm{A}(i_2, j_2) = 1$ and $|j_1 - j_2| < I(i_1, i_2)$, then we have a contraint violation.
More precisely,

$$V(A) = \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{k=1}^{n} \sum_{l=1}^{n} = \begin{array}{ll} 1 & \text{if } |k-l| < I(i,j) \\ 0 & \text{if not} \end{array}$$

By minimizing the V(A) operation, we get a matrix A that describes the color distribution is such a way that the number of violations to the constraints are minimized.

4.5.1 Heuristics

Because solving this combinatorial optimization problem is an NP-Complete Problem, computer scientists and engineers have developed heuristic methods to help solve the channeling allocation problem.

Simulated Annealing
This heuristic model comes directly from thermodynamics, especially the way liquids freeze and crystallize. The idea is to create a procedure to solve the optimization problem by sampling the possibilities in such a way that it has a high probability of finding optimal or near-optimal solutions in a reasonable time [6]. Usually the heuristic creates a *local neighborhood* or the set of all possible states after making a slight change to the state. For instance in channeling allocation, a slight change may be swapping the needs of two cells [rows of matrix A]. By doing this, you preserve the frequency demands. In addition, experiments have shown that randomly choosing the cells to switch provides better results over selecting more than one cell at a time or cycling among the seven cells [7]. Determining the states to sample is through a probability distribution based on annealing, specifically the rate at which temperature decreases.

Tabu Search
This is one of the most commonly used heuristics. It prevents cycling by penalizing moves to previously visited locations in the search space. The Tabu Search utilizes prior search history in addition to the lowest cost. Two basic components make up the Tabu Search: *short-term and long-term* memory [7]. Essentially instead of a A matrix as described above, a cube would be used to keep track of the moves and hence able to restrict any repeat moves.

4.6 Implementation

In a realistic case, our system of equations become too large to obtain a feasible solution in a reasonable time. In our problem, we have over 3000 different F-T pairs, 50 cells, and 40 people per cell which means our matrix I will be a

50×50 matrix and A will be a 50×3000 matrix. The constaint matrix T will have entries of 42 which allows for even more combinations of A which implies that the space of possible repeater distributions would be extremely large. This makes our theoretical method to get the most optimal solution impractical.
We thus simplify the problem in the cost of accuracy in the following way.

- The group of F-T pairs per cell is represented by one color
- The repeaters in a cell is located at the center of the cell
- We use a random number generator to create our A matrix and choose the matrix A such that the number of violations is minimized after a certain amount of runs.

4.6.1 Plan of attack

We first index each cell with a natural number and create our matrix I which encodes our weak constraint that attempts to limit the amount of co-channel interference.
In the case of 10,000 simultaneous users, there will be much more entries of 1 than in the 1,000 case.
Instead of keeping the color of adjacent cells different like in the example, we instead contrain the colors of cells within the reuable distance of 50 miles of each other to be of different colors.
This may be impossible in some cases so we weakly constain it and make the number of violations to the constraint minimized.
We then draw circles centered at the ith cell of radius 50 miles and for every jth cell within that reusable distance, we input $I(i, j) = 1$ and for those outside, a 0.
Once we've constructed the matrix I to fit our scenario, we set up our program to create 100×3000 matrices with a random input generator. With each created matrix, the program determines the number of violations to the weak constraint. By running this program over and over again, we obtain a matrix A that violates the constaints minimally and thus have found our obtimum distribution.

4.6.2 Pseudocode

```
Input Constraint Matrix
Generate Potential Distribution Matrix
    For all entries of Distribution Matrix
        Generate Random Number random number = 0 or 1
            If row of Distribution Matrix sums to less than 1
                then place random number in the entry of Distribution Matrix
            Else place a 0
        Continue until Distribution Matrix is complete
    end
```

```
If The rows of Distribution Matrix each sum up to 1
    then Accept Distribution Matrix
        For all combination of indicies of Distribution Matrix
            If entries of Distribution Matrix = 1
                then Check contradiction and sum up all contradictions
        end
    If minimum so far > new distribution matrix sum
        then save new sum as minimum and
            new distribution matrix
Else Reject Distribution Matrix

Restart Process of Generating Distribution Matrix
```

4.7 Mountains

We started with the assumption that everything is based on a flat surface and the propagation of the radio waves is uniform; meaning the signal sent by a transmitter at one location travels just as far as signals sent in another. In the case with a uneven terrain, i.e., mountains, we must consider the larger effects of radio wave propagation. To describe the propagation loss of radio waves we used an equation derived from emperical data which is commonly used by the International Telecommunications Union (ITU),

$$\Delta P = 40 \log(d_m) - 20 \log(h_T h_R),$$

[1] where ΔP is the difference between power transmitted and power recieved, d_m the distance between the transmitter and reciever in meters, and $h_T h_R$ is the product of the height of the transmitter and the height of the reciever in meters.

4.8 A more complicated terrain

Radio wave signals can only propagate and arrive at a receiver if the receiver is in the "line of sight" of the transmitter. When the terrain is more complicated, for example mountainous, the "line of sight" becomes non-uniform and location dependent. This adds another dimension to the problem in that placement of the user and the repeater must be taken into account. Repeaters on mountaintops send a farther signal making the signal more prone to co-channel interference however, by the propagation equation introduced above, the user signals are more easily accepted. Conversly, repeaters in a valley reduce co-channel interference, but the user must be much closer.

Our model doesn't account for this irregularity and thus must be altered to account for such conditions.

- The matrix I which encodes our weak constraints must be altered and weights must be added, i.e. entries of 2, 3, or 4, etc. depending on the importance of each constraint to again find the most efficient repeater distribution.
- Our probability distribution was assumed to be uniform since one part of the terrain was not any more favorable than any other part of the terrain. In the case of mountains, we would have to take into account the relative population of inhabitants in mountains versus the ground. Thus implimenting a non-homogeneous Poisson process.

5 Results

The problem of minimizing repeaters in a given area while providing simultaneous coverage to all users proved to be multifaceted:

- Based on our assumptions about the usability of repeaters, we determined that for each population case, the **lower bound of the number of necessary repeaters was the number of users**.
- In order to provide an accurate approximation of how transmitter signals would be distributed throughout the region, we then required a model of probable user population density in our network area. Using an upper tail bound Poisson process, we determined that **the upper bound of users**, and thus necessary repeaters, within any 5 mile radius in the network area was **42 for the 1,000 user case, and 249 for the 10,000 user case**.
- Mapping our network area as a circular plane with a lattice covering of regular hexagons, we modeled our user population at its upper bound by assigning each cell its maximum number of users.
- This method used the distribution matrix, A, to minimize interference, and thus determine the most efficient placement of repeaters.

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