Math 336 Sample Problems

One notebook sized page of notes will be allowed on the test. The test will cover up to §V.7 in the text (excluding those sections for which there was no homework). The midterm will be on Monday, May 14.

1. Let f be analytic on the connected open set W. Suppose $\{z : |z-a| \leq d\} \subset W$ and suppose f is real on $\{z : |z-a| = d\}$. Prove that f is constant in W.

2. Let f be an analytic function on an open connected set W. Suppose $0 \in W$ and suppose $|f(\frac{1}{n})| < e^{-n}$ for all $n > 0$. Prove that $f(z) = 0$ for all $z \in W$.

3. Suppose $f(z)$ is an entire function and $|f(z)| < 1 + |z|^{1/2}$. Prove that f is constant.

4. Suppose that f is analytic and non-constant on the disk $\{|z - z_0| < R\}$. Suppose $Re(f(z_0)) = 0$, Prove that on every circle $\{|z - z_0| = r\}$, with $0 < r < R$, $Re(f)$ assumes both positive and negative values.

5. Suppose f is continuous on a connected open set W. Suppose also that f^2 and f^3 are analytic on W. Prove that f is analytic.

6. Let u and v be harmonic on an open connected set W. Suppose that $u(z)v(z) = 0$ on an open subset of W. Prove that either u or v is identically 0 on W.

7. Suppose $f(z) = u(z) + iv(z)$ is entire and $|u(z)| > |v(z)|$ for all z. Prove that f is constant.

8. Where does
$$\sum_{n=0}^{\infty} e^{-z^2\sqrt{n}}$$
converge? Where is it analytic?

9. Suppose $\text{Re}(z_1) < 0, \text{Re}(z_2) < 0$. Prove that
$$|e^{z_1} - e^{z_2}| < |z_1 - z_2|.$$

10. Suppose f is entire and $|f(z)| \leq |Ke^z|$ for some K. Prove that $f(z) = Ce^z$ for some C.

11. Suppose $f(z) = u(z) + iv(z)$ is analytic in all of $\mathbb{C}$. Suppose also that $u(z)v(z) \geq 0$ for all $z \in \mathbb{C}$. Prove that f is constant.

12. Let $g(z)$ be a function that is analytic at a and suppose that the series

$$g(z) + g'(z) + g''(z) + g'''(z) + \ldots,$$

converges at $z = a$. Prove that g is analytic in all of $\mathbb{C}$ and that the series

$$g(z) + g'(z) + g''(z) + g'''(z) + \ldots,$$

converges uniformly on compact subsets of $\mathbb{C}$. Hint: Use that fact that

$$g(a) + g'(a) + g''(a) + g'''(a) + \ldots,$$

is Cauchy.

13. There may be homework problems, example problems from the text, or requests for statements of theorems on the midterm.