Optimal Drafting in Professional Sports

Dylan Austin, Math 336
May 22nd, 2015

1. Introduction

Each of the major professional sports leagues holds an annual draft for teams to select new talent from high school and college. In its simplest form, an order of teams is determined which is then followed each round, during which each team will select one player, until a certain number of rounds is completed. Each player has a different value to each team, so it's most likely that every team will have its own preferences of players. In their paper "Prisoners' Dilemma and Professional Sports Drafts," Steven Brams and Philip Straffin, Jr. discuss the optimization of such a draft and how this relates to the famous, paradoxical Prisoners' Dilemma [1].

2. Basics of Optimization

Optimization is the process by which a group attempts to maximize its outcome in some situation. In this case, a team would be optimizing its draft by getting players that are high on its preference list. In reality, drafting is much more complex, involving trades, money, and intense information gathering. However, so that these drafts can be analyzed in a simpler way, these are all ignored in favor of some simple assumptions made by Brams and Straffin [1]:

a) Strict Preferences: Each team's list of preferences is strict, there are no "ties" between players, so that when choosing between players x_1 and x_2, the team will always prefer the same player.

b) Partial Ordering: These players are then put in order of preference when they are picked (which may not be in order of preference), and one group of players is only considered better if in each slot, the player is the same or more preferred than the corresponding player in the other group. This is called pairwise comparison.

c) Self-Interest: Each team is only interested in maximizing their draft, and not in preventing other teams from doing so.

d) Independence: Each team acts individually – no deals are made between teams.

e) Complete Information: The full preference list for each team is available to all teams, and decisions can be made based on this knowledge.

All of these fail in one way or another during an actual draft, but such assumptions are necessary in order to find the true optimal strategy for each team in a draft. This leads to the working definition of optimization is used by Brams andStraffin: Pareto Optimality.

Definition: An outcome S is said to be Pareto Optimal if there is no other outcome S' for which all teams have the same players or prefer S' by pairwise comparison [1].

This is not to say that there are multiple Pareto optimal outcomes to a draft; in a simulation below this will be shown to be possible. It is also not to say that a Pareto optimal outcome is produced by optimal choices, as one of the results of the paper is that the two are only the same in the 2-team case.

3. The Prisoners' Dilemma

Before proceeding to draft simulations and formulations, it's best to cover the famous Prisoners' Dilemma. It is set up as such: two people, A and B, have been caught by the police for the same crime. They are interrogated in separate rooms, and can choose to either stay silent and protect the other, or confess that they committed the crime. If one confesses and the other stays silent, the confessor goes free and the silent prisoner receives a harsh sentence. If both confess, this sentence is split between the two of them. But if both stay silent, they can't be charged and so receive minimal jail time [1]. An example grid for time served is shown below:

(A's time served, B's time)	B stays silent	B confesses
A stays silent	(1 year, 1 year)	(10 years, 0 years)
A confesses	(0 years, 10 years)	(5 years, 5 years)

It's not hard to see from the above table that regardless of B's choice, A will serve less time by confessing, and regardless of A's choice, B will serve less time by confessing. Since both prefer to serve as little jail time as possible, the logical choice is for each of them to confess. But this is not the Pareto optimal choice, as mutual silence is a better choice for both of them than mutual betrayal. Thisat a glance seems like a paradox – each person made the best choice, but ended up with an outcome that is strictly worse than another. Though this dilemma is mostly studied in its original two-person form, it can be extended to any number of people. Komorita does this by having two linear functions for those who stay silent and those who confess, which both increase as more people confess but so that it is always optimal for someone to confess [3]. The game should end the same way, with everyone choosing to confess and resulting in them all having a sentence that is longer than if they just all stayed silent. Once more, the optimal strategy does not produce a Pareto optimal solution. As a note, the Prisoners' Dilemma is not a standard which the draft will follow, considering the draft has outcomes that vary based on teams' preferences, whereas the dilemma does not.

4. The Two-Team Draft

So now we tackle the two team draft. Let's take a situation with four players, 1 through 4, with teams A and B such that their preferences are shown below from left to right:

A: 3 2 1 4
B: 2 4 1 3

A team following a sincere strategy will always pick their most preferred player left. If both teams follow a sincere strategy here, the result will be as follows, A will pick 3, B will pick 2, A will pick 1, and B will pick 4. We'll denote this result as ?$\begin{matrix} 3 & 1 \\ 2 & 4 \end{matrix}$?. Note that this result is Pareto optimal, as B got its first two preferences, and in order for A to improve its choices it would have to take player 2 away from B. However, there is another way for this draft to proceed, and that is by making optimal choices. A, instead of picking player 3 first, will select player 2, followed by B picking player 1, A picking 3, and B picking 4. This result is ?$\begin{matrix} 2 & 3 \\ 1 & 4 \end{matrix}$?, and is the best outcome A can produce, so team A should take it. This outcome is worse for B than before, but B cannot prevent A from taking player 2 first, so this is B's optimal play. This result is also Pareto optimal, as A has their first two preferences and in order for B to improve they would have to take player 2 away from A. This outcome is referred to as the sophisticated outcome, which results when all teams make optimal decisions.

This was just an example; in an arbitrary two-team draft with any number of players, the optimal decisions are determined by the Kohler-Chandrasekaran method [1]. In it, the draft is worked backwards, beginning with team B's last choice, which is the player last on player A's list. Team A's last choice is then the lowest player on B's list after the first player has been removed. Then team B's second to last choice is the player last on player A's list after these first two players have been removed, and so on and so forth, until the draft reaches player A's first choice. Applying that to the above, we see that it does produce the sophisticated outcome again: B will pick 4 last, A will pick 3 last, B will pick 1 first, and A will pick 2 first, once again ?$\begin{matrix} 2 & 3 \\ 1 & 4 \end{matrix}$?.

Theorem: The sophisticated outcome from the Kohler-Chandrasekaran method for two teams will always produce a Pareto optimal outcome [1].

Proof: Let S be the sophisticated outcome, and assume it is not Pareto optimal such that there is another outcome S' such that each team prefers S' to S, as neither team can have the exact same players. Then, among the players each team receives in S' and not in S, they have a most preferred player, call them player X for team A and player Y for team B. So, because swapping these players improves the outcome for both teams, team A prefers X to Y and team B prefers Y to X, but in outcome S team A gets Y and team B gets X.
It follows then from the Kohler-Chandrasekaran method that in the sophisticated outcome, because X is preferred to Y for team A, Y would have been assigned to as a choice for B first unless it was assigned as a choice for A. So Y must have been assigned as a choice for A before X was assigned to B. But team B prefers Y to X, so X would have been assigned as a choice for A first unless it was assigned as a choice for B. So X must have been assigned as a choice for B before Y was assigned to A. This is a contradiction, so there cannot be an S' that is pairwise comparison better than S. So outcome S is Pareto optimal [1].

This shows that in the case of two teams, a Prisoners' Dilemma situation where optimal choices lead to a non-optimal outcome doesn't exist. If both teams have complete information and choose accordingly, only a Pareto optimal outcome is possible. However, an investigation of the three team case will show that this does not hold.

5. The N>2 Team Draft

Let’s examine another sample draft, this time with three teams and six players, whose preferences are as follows [1]:

A:	1	2	3	4	5	6
B:	5	6	2	1	4	3
C:	3	6	5	4	1	2

The sincere outcome is easy to see, and is ?$\begin{matrix}1 & 2\\ 5 & 6\\ 3 & 4\end{matrix}$?. A quick check reveals that this result is Pareto optimal, as teams A and B each get their first two preferences, and in order for C to improve their outcome, they’d have to take player 5 or 6 away from B. It is in fact not hard to show that by making sincere choices, a Pareto optimal outcome is guaranteed.

Theorem: A sincere outcome is always Pareto optimal [1].

Proof: Let S be the sincere outcome, determined by all teams choosing the player they prefer the most of those remaining, and let S’ be any other outcome. Denote the set of players who are chosen by different teams in S and S’ as P, and let the first chosen player in S among this group. He was chosen by team A, so team A must prefer him to all other players in P, one of which they instead chose during S’. So by pairwise comparison, S’ cannot be a better outcome for team A than S is. This applies to any S’, so S is Pareto optimal [1].

This is interesting – if every team simply chooses their most preferred player, then there is no way to improve upon every team’s result. This is not to say though, that individual teams cannot attempt to improve upon their choices, such as in the two-team case above, where team A’s methods gave them a better outcome.

The first team that wants to change this is C, as they get their first and fourth choices. If they instead took player 6 in the first round, the draft would go as such: ?$\begin{matrix}1 & 2\\ 5 & 4\\ 6 & 3\end{matrix}$?, and now they have their first two preferences. But now team B is stuck with their first and fifth choices. If they were to choose player 6 in the first round, the result would be $\begin{pmatrix} 1 & 2 \\ 6 & 4 \\ 5 & 3 \end{pmatrix}$, which is still bad for them, but if they instead anticipate C's move by choosing player 2 in the first round, leading to $\begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}$, which is good for teams B and C, but not for team A, which now has their first and fourth pick. So they can instead pick player 3 first, which regardless of B's choice of 5 or 2 leads to $\begin{pmatrix} 3 & 1 \\ 5 & 2 \\ 6 & 4 \end{pmatrix}$. Indeed, this is the result if all teams pick optimally [1]. But there is a problem with this in that it is not Pareto optimal – the sincere outcome is actually better for all teams. Team A prefers 1 and 2 to 1 and 3, team B prefers 5 and 6 to 5 and 2, and team C prefers 3 and 4 to 6 and 4. This is a proof by example of the following theorem:

Theorem: With three or more teams, optimal play may lead to an outcome that is not Pareto optimal [1].

This is the result that is just like the paradox of the Prisoners' Dilemma. By attempting to improve their individual outcomes, these teams have only succeeded in making it worse for everyone. If they all drafted "sub-optimally" by following their sincere preferences, they would have come out better. This occurrence even happens in the same way as in the Prisoners' Dilemma. In it, should anyone choose silence, the other prisoners can take advantage of their silence and punish them for it. And here, should any team attempt to make a sincere pick, they can be punished by the other two teams undercutting them. And so in order to avoid this, the teams force themselves into a situation where they all suffer.

6. Alternatives

Of course, in real life this type of drafting disaster is commonly avoided through various means, the first being trading. Even in the simplest of forms, it can improve these drafts. Take the above sophisticated solution as an example. It can be reverted to the sincere solution by a three-way trade that sends player 2 to team A, player 6 to team B, and player 3 to team C, thus returning to a Pareto optimal solution [1]. It is also true that for the above example, team A moving back to the third pick, giving team B the first and team C the second, causes the sophisticated outcome to be the same as the sincere outcome, which is ? $\begin{matrix}5 & 6\\ 3 & 4\\ 1 & 2\end{matrix}$? for ? ? ?, the same as above [1]. Another way to achieve a better outcome is by communication, which is not possible in the classic Prisoners' Dilemma, but has been shown Komorita in a larger game to play a role. In some situations, a coalition of some, but not necessarily all, of the prisoners can overcome the potential for betrayal [3]. Applying it to the draft, a deal made between enough teams could force the sincere outcome through and prevent other teams from using optimal play to punish sincere picks.

In reality, professional sports drafts have evolved far beyond any of this. Featuring 30 or more teams, up to 7 rounds, and hundreds of players, attempting to apply these simple tactics is futile. Instead, far more complex analysis is used, such as that by El-Hodiri and Quirk, which use numerous variables to measure the economic impact of every move a sports team makes. It evaluates players based on units of playing skills they would contribute to the team alongside the economic impact that would come both from their salary and their performance, which vary depending on from where they are acquired [2]. But despite all this, the dilemma can persist, an ever present reminder of what can go wrong when we try to make things right.

Sources

[1] Bram, Steven J. and Philip D. Straffin, Jr. "Prisoners' Dilemma and Professional Sports Drafts." *The American Mathematical Monthly* 86.2 (1979): 80-88. *JSTOR*. Web. 19 May 2015.

[2] El-Hodiri, Mohamed and James Quirk. "An Economic Model of a Professional Sports League." *Journal of Political Economy* 79.6 (1971): 1302-1319. *JSTOR*. Web. 21 May 2015.

[3] Komorita, S. S. "A model of the N-Person Dilemma-Type Game." *Journal of Experimental Social Psychology* 12.4 (1976): 357-373. *Science Direct.* Web. 19 May 2015.

http://www.jstor.org.offcampus.lib.washington.edu/stable/2321942?Search=yes&resultItemClick=true&searchText=sports&searchUri=%2Faction%2FdoBasicSearch%3FQuery%3Dsports%26amp%3Bfilter%3Djid%253A10.2307%252Fj100069%26amp%3BSearch%3DSearch%26amp%3Bwc%3Don%26amp%3Bfc%3Doff%26amp%3BglobalSearch%3D%26amp%3BsbbBox%3D%26amp%3BsbjBox%3D%26amp%3BsbpBox%3D&seq=5#page_scan_tab_contents

http://www.jstor.org.offcampus.lib.washington.edu/stable/1830103?seq=5#page_scan_tab_contents

http://ac.els-cdn.com/S0022103176800042/1-s2.0-S0022103176800042-main.pdf?_tid=0a7543c0-fe8d-11e4-8423-00000aacb362&acdnat=1432084424_ab41ca4eff72e2f32f1dd0d2b00f0a24