

## Math336 Midterm, May 20, 2013

Name: \_\_\_\_\_

One notebook sized page of notes is allowed.

1. Using Rouché's theorem, show that  $z^7 + 5z^3 + z - 2$  has four roots in the set  $\{z : 1 < |z| < 2\}$ .

2. Let  $f$  be an analytic function on an open connected set  $W$ . Suppose  $0 \in W$  and suppose  $|f(\frac{1}{n})| < e^{-n}$  for all  $n > 0$ . Prove that  $f(z) = 0$  for all  $z \in W$ .

3. Let  $f(z) = e^{z-1/z} = \sum_{n=-\infty}^{n=+\infty} a_n z^n$ . Prove that

$$a_n = \frac{1}{2\pi} \int_0^{2\pi} \cos(2 \sin(\theta) - n\theta) d\theta.$$

4. Using a contour integral prove

$$\int_0^{2\pi} \frac{dt}{1 + \sin^2 t} = \sqrt{2}\pi.$$