Math 336 $\mathfrak{Midterm}$, April 29, 2013

Name:____

One notebook sized page of notes is allowed on the test.

1. Let $f(z) = u(z)+iv(z), u = Re(f(z)), v = Imf((z))$ be analytic on an open connected set Ω. Suppose there are real numbers a, b, c with $a^2 + b^2 \neq 0$ so that $au(z) + bv(z) = c$ for all $z \in \Omega$. Prove that f is constant.

2. Suppose u is harmonic on $\mathbb{C}$. Prove that if $u(z) \to 0$ as $|z| \to \infty$ then $u(z) = 0$ for all z.

3. Compute

$$\int_{|z|=1} \frac{dz}{(2z-1)(z-2)}.$$

4. Let $f(z) = e^{-z^{-4}}$ if $z \neq 0$, $f(0) = 0$. Prove that f is analytic at z if $z \neq 0$ and that the Cauchy-Riemann equations are satisfied at 0. Is f analytic at 0?