$\mathfrak{Math336}$ Midterm, May 21, 2012

Name:_____

One notebook sized page of notes is allowed.

1. Using Rouché's theorem, show that $z^7 + 5z^3 + z - 2$ has three roots in the set $\{z : |z| < 1\}$.

2. Let f be an analytic function on an open connected set W. Suppose $0 \in W$ and suppose $|f(\frac{1}{n})| < e^{-n}$ for all $n > 0$. Prove that $f(z) = 0$ for all $z \in W$.

3. Suppose f is analytic on the disk $D = \{z : |z| < 1\}$ and suppose f is continuous on the closure of D. Suppose

$$|f(e^{it})| \leq R_j,$$

when $(j-1)\frac{\pi}{2} < t < j\frac{\pi}{2}$ for $j = 1, 2, 3, 4$. Prove

$$|f(0)|^4 \leq R_1 R_2 R_3 R_4.$$

4. Using a contour integral compute

$$\int_{-\infty}^{\infty} \frac{dx}{x^4 + 1}$$