Math 336 𝔐𝔦𝔡𝔱𝔢𝔯𝔪, April 30, 2012

Name:‾‾‾

One notebook sized page of notes is allowed on the test.

1. Let $f : \mathbb{C} \to \mathbb{C}$ be defined by $f(z) = x^3 y^2 + i x^2 y^3$. Find the points where f is complex differentiable and the points where f is complex analytic.

2. Suppose u is harmonic on $\mathbb{C}$ and $u(0) = 0$. Prove that if $u(z) \to 0$ as $|z| \to \infty$ then $u(z) = 0$ for all z.

3. Compute

$$\int_{|z-1|=2} \frac{\sin(z)dz}{(z-4)(z-1)}.$$

4. Suppose f is analytic on $D = \{z : |z| < 1\}$ and continuous on the closure $\overline{D}$ of D. Suppose $f(z) \neq 0$ in D. Prove that the minimum of $|f(z)|$ on $\overline{D}$ occurs on the boundary of D.