Green's Functions and Their Applications to Quantum Mechanics

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1 Introduction

The intent of this paper is to give the unfamiliar reader some insight toward Green's functions, specifically in how they apply to quantum mechanics. I plan to introduce some of the fundamentals of quantum mechanics in a rather unconventional way. Since this paper is meant to have a stronger focus on the mathematics behind Green's functions and quantum mechanicical systems, as opposed to the physical interpretation, I will not place much of an emphasis on the postualates of quantum mechanics, with the exception of one which fits in nicely with the discussion of Hilbert Spaces. Quantum mechanics and Green's functions, at first glance, seem entirely unrelated, however within the last 50 years Green's functions have proven themselves to be a useful tool for solving many flavors of boundary value problems within the realm of quantum mechanics. In addition to this, Green's functions have proven to play a large role in many body theory, perturbation theory, and even in the development of modern quantum mechanics.

Section 2 of this paper is meant to serve as an introduction to the linear algebra behind quantum mechanics. This section will not contain any information about Green's functions, and is meant to develop the necessary machinery for solving basic problems in quantum mechanics. I plan to show how working in dual Hilbert spaces makes working with quantum mechanical operators very convenient. I will also introduce Hermitian operators and the general properties they hold.

Section 3 will be almost entirely dedicated to Green's functions in quantum mechanics. The section will begin with deriving and stating some useful properties of time independent Green's functions. Much of the information that will be presented in this section will be from E.N. Economou's *Green's Functions in Quanatum Mechanics*, [1] which is a very advanced text. I hope to elucidate some of the jumps he assumes an astute reader with a strong physics background will make, in order to make the material a bit more accessible to an undergraduate mathematics student. After discussing the time independent Green's functions, I plan on showing the true power of the Green's function method by solving both the time independent and time dependent Schrödinger equation using Green's functions.

2 Linear Algebra

Linear algebra plays a significant role in quantum mechanics, specifically with the concept of quantum states, which are the mathematical variables that describe a quantum system. This section is to serve as an introduction to bra-ket notation, and to develop the necessary background to both solve problems and prove theorems in quantum mechanics using this very powerful and convenient notation. In the first subsection, I give a definition for both a bra and a ket that uses material which will be explained later in this section. Though this seems a bit convoluted, I decided it would be best to give the reader a preview as to what we will be talking about in subsequent subsections that makes bra-ket notation so powerful.

2.1 Introduction to Bra-Ket Notation

We will begin our introduction to bra-ket notation by defining our ket, $|i\rangle$ as a vector in a continuously infinite dimensional complex Hilbert space of complex square integrable functions on the reals. We define our bra, $\langle i|$ as the complex conjugate vector of $|i\rangle$, residing in the dual space of the Hilbert space $|i\rangle$ is contained in. It is often convenient to think of a ket as a column vector and a bra as a row vector. With this, we can see that the outer product, $|i\rangle \langle j|$ will form some infinite matrix $\mathbf{X}$, which can act as on operator transforming a ket to another ket. In the following subsections, we will briefly explain the notion of Hilbert spaces, L^2 (square integrable) spaces, completeness, and of course, operators.

2.2 Vector Spaces

A **vector space** V is a collection of vectors $|i\rangle$ satisfying the following properties: [5]

- $|i\rangle + |j\rangle = |k\rangle$, where $|k\rangle$ is a unique vector in V.
- All vectors in V commute, that is, $|i\rangle + |j\rangle = |j\rangle + |i\rangle$.
- Associativity holds; $(|i\rangle + |j\rangle) + |l\rangle = |i\rangle + (|j\rangle + |l\rangle)$.
- There exists some vector $|\mathcal{O}\rangle$ such that $|\mathcal{O}\rangle + |i\rangle = |i\rangle$ for every $|i\rangle$.
- For every $|i\rangle \in V$, $-|i\rangle$ is also in V and $|i\rangle + (-|i\rangle) = |\mathcal{O}\rangle$.
- Scalar multiplication follows such that,

$$a(b\,|i\rangle) = (ab)\,|i\rangle$$

$$\mathbf{1}\,|i\rangle = |i\rangle$$

$$a(|i\rangle + |j\rangle) = a\,|i\rangle + a\,|j\rangle$$
$$(a+b)\,|i\rangle = a\,|i\rangle + b\,|i\rangle$$

Where a and b are complex constants.

- Let V, W be vector spaces of a field, $F \in \mathbb{C}$. If $V \subset W$, then V is a **subspace** of W.

Let $I = |i_1\rangle, ..., |i_n\rangle$ be a set of linearly independent vectors in V. If $dim(V) = n$; then I is a basis for V. Note: n can either be finite, countably infinite, or uncountable infinite. The following theorem will be stated without proof:

Theorem 2.1. If I forms a basis for V, then any vector $|j\rangle$ can be expanded in the basis as $|j\rangle = \sum_k a_k\,|i_k\rangle$.

For a vector space $V \in \mathbb{C}$, we define the dual space, V^* as:

$$V^* \equiv \langle j| : V \to \mathbb{C} \tag{2.1}$$

Given the inner product space $\langle j|\,i\rangle$, and given that $|i\rangle \in V$ and $\langle i| \in V^*$ we can construct the following isomorphism:

$$|i\rangle \leftrightarrow \langle i| \tag{2.2}$$

$$V \leftrightarrow V^* \tag{2.3}$$

$$a\,|i\rangle \leftrightarrow \bar{a}\,\langle i| \tag{2.4}$$

2.3 Inner-Product Spaces, Hilbert Spaces, and L^2 Spaces

A vector space $V \in \mathbb{C}$ is an inner product space if given $|i\rangle, |j\rangle \in V$, there is an inner product $\langle i|\,j\rangle \in \mathbb{C}$ satisfying: [5], [6]

$$\langle i|\,j\rangle = \overline{\langle j|\,i\rangle} \tag{2.5}$$

$$\langle i|\,(|j\rangle + |j'\rangle) = \langle i|\,j\rangle + \langle i|\,j'\rangle \tag{2.6}$$

$$\langle i|\,(a\,|j\rangle) = a\,\langle i|\,j\rangle \tag{2.7}$$

$$\langle i|\,i\rangle \geq 0 \tag{2.8}$$

$$\langle i|\,i\rangle = 0 \Leftrightarrow |i\rangle = 0 \tag{2.9}$$

Definition 1. A space V is complete if every Cauchy sequence, $|i_n\rangle$ converges in V.

Remark. In a complete space, we can define the infinite identity matrix (operator) as: [2]

$$I = \int |i\rangle \langle i| \, d\tau \tag{2.10}$$

Using the notion of completeness, we can now define what is known as a **Hilbert Space**:

Definition 2. An inner-product space that is complete under the metric induced by the inner product is a Hilbert Space, $\mathcal{H}$.

Theorem 2.2. Any complete subspace of an inner-product space is closed. A Hilbert space is a subspace of a Hilbert space if an only if it is closed.

Proof: Lets begin with proving the first statement. Let V be a complete subspace of an inner-product space W. Let x_n be a sequence in V such that x_n converges to $x \in W$. Then x_n is Cauchy in V, and since V is complete, x_n must converge to an element of V, so $x \in V$, hence V is closed. Now we will prove the second statement. Given that a Hilbert space is complete, lets assume that a subspace $R \in \mathcal{H}$ is complete. Then, from the first statement, we know that R is closed. Now suppose R is closed and let x_n be Cauchy in R. Since x_n is Cauchy in $\mathcal{H}$, it must converge to some $x \in \mathcal{H}$. Since R is closed, we have x_n converges to $x \in R$, hence R is closed. A subspace of an inner-product space is also an inner-product space, and we just proved that R is complete, therefore R is a Hilbert space if and only if it is closed. □

Proposition 1. *A Hilbert Space can be either a finite dimensional basis, countably infinite dimensional basis, or uncountably infinite dimensional basis.*

Definition 3. For $p > 1$, let L^p be the set of all sequences $\mathbf{x} = x_n$ of real or complex numbers that satisfy: [4]

$$\sum_{n=1}^{\infty} |x_n|^p < \infty \tag{2.11}$$

We define the p-norm of $\mathbf{x}$ by:

$$\|\mathbf{x}\|_p = \left(\sum_{n=1}^{\infty} |x_n|^p \right)^{\frac{1}{p}} \tag{2.12}$$

Then L^p is a metric space, under the metric:

$$d(\mathbf{x}, \mathbf{y}) = \|\mathbf{x} - \mathbf{y}\|_p = \left(\sum_{n=1}^{\infty} |x_n - y_n|^p \right)^{\frac{1}{p}} \tag{2.13}$$

We can now show that an L^2 space is a Hilbert space.

Theorem 2.3. For $p > 1$, an L^p space is a Hilbert Space only when $p = 2$.

Proof: We see that the inner product, $< \mathbf{x}, \mathbf{y} >= \sum_{n=1}^{\infty} x_n \bar{y_n}$ has a metric;

$$d(\mathbf{x}, \mathbf{y}) = \|\mathbf{x} - \mathbf{y}\|_2 = \left(\sum_{n=1}^{\infty} |x_n - y_n|^2 \right)^{\frac{1}{2}}$$

This agrees with the definition of an L^p space when $p = 2$. An L^2 space is closed and therefore complete, so it follows that an L^2 space is a Hilbert space. When $p \neq 2$, the L^p space is not an inner-product space and is therefore not a Hilbert space. □

Example 1. Let $\psi_1(x)$ be defined on a Hilbert space, $\mathcal{H}$, and $\psi_2(x)$ be defined on it's dual space $\mathcal{H}^*$. We can now definte the inner product in Bra-Ket notation as follows:

$$\langle \psi_2 | \psi_1 \rangle = \int_{-\infty}^{\infty} \langle \psi_2 | x \rangle \langle x | \psi_1 \rangle \, dx \tag{2.14}$$

Since a Hilbert space is complete, it follows that $\int_{-\infty}^{\infty} |x\rangle \langle x| \, dx = I$. This gives us:

$$\langle \psi_2 | \psi_1 \rangle = \int_{-\infty}^{\infty} \langle \psi_2 | x \rangle \langle x | \psi_1 \rangle \, dx = \int_{-\infty}^{\infty} \langle \psi_2 | I | \psi_1 \rangle \, dx = \int_{-\infty}^{\infty} \overline{\psi_2}(x) \psi_1(x) dx \tag{2.15}$$

With the notion of L^2 spaces being Hilbert spaces, we can now introduce the concept of orthonormal bases. An orthonormal basis is a basis $|\psi_i\rangle$ which satisfies, $\langle \psi_i | \psi_j \rangle = \delta_{ij}$, where δ_{ij} is the Kronecker Delta function:

$$\delta_{ij} = \begin{cases} 1 & : i = j \\ 0 & : i \neq j \end{cases} \tag{2.16}$$

Let $|i_1\rangle, |i_2\rangle, ..., |i_3\rangle$ be a basis. The Gram-Schmidt orthogonalization process works as follows. Let

$$|\phi_1\rangle = \frac{|i_1\rangle}{\sqrt{\langle i_1 | i_1 \rangle}} \tag{2.17}$$

Extending this process, we get:

$$|\phi_n\rangle = \frac{|i_n\rangle - \langle i_n | i_n - 1 \rangle |i_{n-1}\rangle - ... - \langle i_n | i_1 \rangle |i_1\rangle}{\sqrt{|i_n\rangle - \langle i_n | i_{n-1} \rangle |i_{n-1}\rangle - ... - \langle i_n | i_1 \rangle |i_1\rangle}} \tag{2.18}$$

This makes $|\phi_i\rangle$ an orthonormal basis. Furthermore, since $|\phi_i\rangle$ is an orthonormal basis, it follows that for all $|i\rangle$

$$|i\rangle = \sum_k a_k |\phi_k\rangle = \sum_k |\phi_k\rangle \langle\phi_k| i\rangle \,, \qquad a_k = \langle\phi_k| i\rangle \tag{2.19}$$

2.4 Linear and Hermitian Operators

A linear operator from a vector space V to another vector space W is a transformation such that

$$A(|i\rangle + |j\rangle) = A|i\rangle + A|j\rangle \,, \qquad \text{for every } |i\rangle \,, |j\rangle \tag{2.20}$$

$$A = B \iff A|i\rangle = B|i\rangle \,, \quad \forall |i\rangle \,. \tag{2.21}$$

Definition 4. Given an operator L, we define the adjoint of L, $L^\dagger$ by:

$$L^\dagger |i\rangle \leftrightarrow \langle i| L \tag{2.22}$$

Definition 5. An operator L is Hermitian if it is self-adjoint, that is, $L = L^\dagger$.

Theorem 2.4. If $L = L^\dagger$ then all eigenvalues, λ_i of L are real, and all eigenstates associated with distinct λ_i's are orthogonal.

Proof:

$$L|\lambda\rangle = \lambda|\lambda\rangle \Rightarrow \langle\lambda| L^\dagger = \langle\lambda| \lambda^\dagger \Rightarrow \langle\mu| (L - L^\dagger |\lambda\rangle) = (\lambda - \bar{\mu}) \langle\mu| \lambda\rangle \tag{2.23}$$

$$\text{if} \quad \lambda = \mu, \qquad \lambda = \bar{\lambda} \quad \text{is real}$$

$$\text{if} \quad \lambda \neq \mu, \qquad <\mu|\lambda>= 0 \qquad \square$$

From this theorem, we can now introduce a fundamental postulate of Quantum Mechanics.

Postulate Observables are Hermitian operators on a self-adjoint Hilbert space.

As you will see in the next section, almost all of the operators we will be dealing with will be Hermitian. The fact that all observables are Hermitian allows us to give physical interpretation to many mathematical constructs.

3 Green's Functions in Quantum Mechanics and Many-body Theory

Now that a brief background of the linear algebra governing quantum mechanics has been discussed, we turn our focus to solving some important boundary value problems that arise in quantum mechanics. To solve these boundary value problems, we will implement the method of Green's functions. One of the most fundamental differential equations governing quantum mechanics is the Schrödinger equation. At the end of the section, we will outline the solution of it using the method of Green's functions. The first subsection will outline the general case of time independent Green's functions. We will not discuss time dependent Green's functions in generality as the math is very complicated. We will, however culminate this section by extending our solution of the Schrödinger to the time dependent case.

3.1 Time Independent Green's Fuctions

We will begin our discussion of Green's functions in quantum mechanics by defining our Green's function in a time independent case.

Definition 6. A Green's function is a solution to an inhomogeneous differential equation of the form: [1]

$$[z - L(r)]G(r, r'; z) = \delta(r - r'), \qquad z \in \mathbb{C} \tag{3.1}$$

This equation is subject to certain boundary conditions for our two position coordinates, r and r' lying on some surface S on the domain Ω of r and r'. We assume $L(r)$ to be a linear, Hermitian, time independent differential operator that possesses a complete set of eigenfunctions $\phi_n(r)$ with eigenvalues λ_n, that is:

$$L(r)\phi_n(r) = \lambda_n \phi_n(r) \tag{3.2}$$

Remark. From our definitions in the previous section, it's not difficult to see that without loss of generality, ϕ_n is an orthonormal basis set, so it satisfies:

$$\int_\Omega \overline{\phi_n}(r)\phi_m(r)dr = \delta_{nm} \tag{3.3}$$

Since $\phi_n(r)$ is complete, we can write

$$\sum_n \phi_n(r)\overline{\phi_n}(r') = \delta(r - r') \tag{3.4}$$

where $\delta(r-r')$ is the Dirac delta function and

$$\delta(r-r') = \begin{cases} 0 & : (r-r') \neq a \\ \infty & : (r-r') = a \end{cases} \tag{3.5}$$

Remark. It is important to note that the Dirac delta function, acting on sume function $f(r')$ satisfies the following property

$$\int_\Omega \delta(r-r')f(r')dr' = f(r), \qquad \text{for some} \quad r' \subset \Omega \tag{3.6}$$

With this in place, we can now show a very neat derivation of our time independent Green's function, using bra-ket notation:

Example 2. Using the Bra-Ket notation developed earlier, we define $|r\rangle$ as the eigenvector of the position operator and write the following:

$$\phi_n(r) = \langle r | \, n \rangle \tag{3.7}$$

$$\delta(r-r')L(r) \equiv \langle r| \, L \, |r'\rangle \tag{3.8}$$

$$G(r,r';z) \equiv \langle r| \, G(z) \, |r'\rangle \tag{3.9}$$

$$\langle r | \, r' \rangle = \delta(r-r') \tag{3.10}$$

$$\int dr \, |r\rangle \langle r| = 1 \tag{3.11}$$

Using this notation, we can now rewrite equations (3.1) to (3.4) as,

$$(z-L)G(z) = 1 \tag{3.12}$$

$$L \, |\phi_n\rangle = \lambda_n \, |\phi_n\rangle \tag{3.13}$$

$$\langle \phi_n | \, \phi_m \rangle = \delta_{nm} \tag{3.14}$$

$$\sum_n |\phi_n\rangle \langle \phi_n| = 1 \tag{3.15}$$

Using equations (3.7) to (3.11), we can now take the $\langle r|$, $|r'\rangle$ matrix element of (3.12) to obtain;

$$\langle r| \, (z-L)G(z) \, |r'\rangle = \langle r| \, 1 \, |r'\rangle = \langle r | \, r' \rangle = \delta(r-r') \tag{3.16}$$

The lefthand side of this equation is given by

$$\langle r| \, (z-L)G(z) \, |r'\rangle = \langle r| \, zG(z) \, |r'\rangle - \langle r| \, LG(z) \, |r'\rangle$$

By (4.9), this can be rewritten as

$$zG(r, r'; z) - \langle r| LG(z) \left|r'\right\rangle$$

Since we are working in a complete space, we can use the infinite unitary operator defined in equation (2.14) to rewrite the left hand side of the equation as

$$zG(r, r'; z) - \int ds \langle r| L |s\rangle \langle s| G(z) \left|r'\right\rangle = zG(r, r'; z) - L(r) \langle r| G(z) \left|r'\right\rangle \tag{3.17}$$

Using the relationship established in (3.9), we can rewrite the entire equation as:

$$zG(r, r'; z) - L(r)G(r, r'; z) = \delta(r - r') \tag{3.18}$$

Factoring out the $G(r, r'; z)$ gives us the identical expression to (3.1), and we are done.

The fact that we can express our time independent Green's function in Bra-Ket space means that the Green's function of position coordinates r and r' is defined on a Hilbert space. Since we have shown that we are working on a Hilbert space, and an L^2 space is a Hilbert space, we are no longer restricted to working in the position, r-space, and we are now able to apply a Fourier transform from $r \subset L^2$ to $k \subset L^2$ and work in momentum, k-space. space, etc.

If all of the eigenvalues of $z - L$ are not equal to 0, then we can solve (3.12) as

$$G(z) = \frac{1}{z - L} \tag{3.19}$$

We can then multiply by the unitary operator from (3.15) to obtain

$$G(z) = \frac{1}{z - L} \sum_n |\phi_n\rangle \langle\phi_n| \tag{3.20}$$

Since $\frac{1}{z-L}$ is a constant, we are able to move it inside the sum. After this, we can apply (3.13) to substitute the eigenvalues of our operator L, which gives us the following relationship

$$G(z) = \frac{1}{z - L} \sum_n |\phi_n\rangle \langle\phi_n| = \sum_n \frac{1}{z - L} |\phi_n\rangle \langle\phi_n| = \sum_n \frac{|\phi_n\rangle \langle\phi_n|}{z - \lambda_n}, \qquad z \neq \lambda_n \tag{3.21}$$

The result is analogous in the r representation. From this representation of $G(z)$, we can see that it is meromorphic with a finite number of poles, which coorespond to the discrete eigenvalues of L. Suppose we want to define $G(r, r'; z)$ at $z = \lambda$. Since $G(r, r'; z)$ has a pole at λ, we will have to define $G(r, r'; \lambda)$ by a limiting procedure. In order to do this, we will have to form a branch cut along certain parts of the real axis. We will do this in the following way:

Definition 7. Let G^+ denote our Green's function defined on $Im(z) > 0$ and let G^- be our Green's function defined on $Im(z) < 0$. We then define G at $z = \lambda$ by:

$$G^{\pm}(r, r'; \lambda) \equiv \lim_{s \to 0^+} G(r, r'; \lambda \pm is), \qquad \text{where } z = \lambda + is \tag{3.22}$$

With this background in place, we will sketch out a rough proof of something that was stated as a fact in E.N. Economou's *Green's Functions in Quantum Physics.*

Theorem 3.1. Let z be a complex variable with real part, λ and imaginary part s. Let $L(r)$ be a linear, Hermitian, time independent differential operator with a complete set of eigenfunctions ϕ_n, and let $u(r)$ be an unknown function that L is operating on. Suppose $f(r)$ is an arbitrary inhomogeneous function that gives us the following differential equation:

$$[z - L(r)]u(r) = f(r)$$

Then, the solution to the differential equation $u(r)$ is given by:

$$u(r) = \begin{cases} \int G(r, r'; z) f(r') dr' & : z \neq \lambda_n \\ \int G^{\pm}(r, r'; \lambda) f(r') dr' + \phi(r) & : z = \lambda \end{cases} \tag{3.23}$$

Proof: We will prove only the case where $z \neq \lambda_n$, the other case is a much more tedious process so we will take it as a fact. We will begin our proof by letting $u(r)$ operate on our proposed solution:

$$[z - L(r)]u(r) = [z - L(r)] \int G(r, r'; z) f(r') dr' \tag{3.24}$$

Both $z - L(r)$ and the integral operator are linear operators, so we are allowed to move $z - L(r)$ inside the integral sign.

$$[z-L(r)]u(r) = [z-L(r)] \int G(r, r'; z) f(r') dr' = \int [z-L(r)] G(r, r'; z) f(r') dr'$$

Using the relationship established in (3.1), we see:

$$\int [z - L(r)] G(r, r'; z) f(r') dr' = \int \delta(r - r') f(r') dr' \tag{3.25}$$

Which by the relationship laid out in (3.6), gives us our desired result. □

Corollary. : A solution, $u(r)$ doesn't exist when z coincides with a discrete eigenvalue of L, unless all of the eigenfunctions associated with λ_n are orthogonal to $f(r)$.

3.2 Solving the Schrödinger Equation Using Green's Functions

The non-relativistic, one particle time independent Schrödinger equation can be written as:

$$H\psi = E\psi \tag{3.26}$$

Or equivalently,

$$[E - H(r)]\psi(r) = 0 \tag{3.27}$$

Where H is the Hamiltonian operator and E denotes the corresponding discrete eigenvalues. The general formalizm developed in the previous subsection can be extended to solving the Schrödinger equation as follows:

$$L(r) \to H(r)$$

$$\lambda \to E$$

$$\lambda + is = z \to z = E + is$$

$$\lambda_n \to E_n$$

$$\phi_n(r) \to \phi_n(r)$$

We'll now define our Green's function for the Schrödinger equation as

$$[E - H(r)]G(r, r'; E) = \delta(r - r') \tag{3.28}$$

Proceeding as we did in our proof of (3.23), we obtain the following result:

$$\psi(r) = \begin{cases} \int G(r, r'; z) f(r') dr' & : z \neq \lambda_n \\ \int G^{\pm}(r, r'; \lambda) f(r') dr' + \phi(r) & : z = \lambda \end{cases} \tag{3.29}$$

Where $G(r, r'; z) = \sum_n \frac{\phi_n(r)\overline{\phi_n}(r')}{z - E_n}$, and $f(r) = 0$. As you can see, the solution to the time independent Schrödinger equation follows directly from the general formalizm of the time independent Green's functions described in the previous subsection. We can now extend this and introduce the time dependent Schrödinger equation as

$$\left(i\hbar\frac{\partial}{\partial t} - H\right)|\psi(t)\rangle = 0 \tag{3.30}$$

Where $\hbar = \frac{h}{2\pi}$ and h is Planck's constant, given in SI units as

$$h \approx 6.626x10^{-34}Js$$

We will now define the time evolution operator as

$$U(t - t_0) \equiv e^{\frac{i(t-t_0)H}{\hbar}}.$$

We can define a Green's function for this operator as follows:

$$U(t - t_0) = i\hbar\tilde{g}(t - t_0) \tag{3.31}$$

Where

$$\tilde{g}(t - t_0) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} e^{-i\omega(t-t_0)}\tilde{G}(\hbar\omega) \tag{3.32}$$

Which can be obtained by means of a Fourier Transform. Now we will write the time dependent Schrödinger equation in terms of the time evolution operator:

$$|\psi(t)\rangle = U(t - t_0)\,|\psi(t_0)\rangle \tag{3.33}$$

We can now rewrite this equation in the r representation to obtain:

$$\psi(r, t) = i\hbar \int \tilde{g}(r, r', t - t_0)\psi(r', t_0)dr' \tag{3.34}$$

4 Conclusion

Solving the one particle non relativistic Schrödinger equation gives only a small glimpse at the true versatility of the Green's function method. In the Schrödinger picture alone, Green's functions serve as invaluable companion for finding solutions to complicated perturbed systems. Green's functions can be used in many physical situations outside of quantum mechanics as well. Some examples include solving Poisson's equation with Dirichlet boundary conditions, solving the classical simple harmonic oscillator, or even the spherical harmonic oscillator. They arise in many situations involving elliptic partial differential equations, and are an effective tool for many boundary value problems. To the reader interested in seeing more applications of Green's functions working outside of quantum mechanics, I suggest looking at G.F. Roach's *Green's Functions* [3]. It provides an introduction to Green's functions working on many different types of differential equations and provides a solid background of using the method in many situations. For the reader interested in learning more about quantum mechanics, I suggest first working through Ira Levine's *Quantum Chemistry* [2], then if you are interested in seeing more applications of the Green's function method within quantum physics I would suggest first glancing over Vladimirov's *Equations of Mathematical Physics* [6] and then go through Economou's *Green's Functions in Quantum Physics* [1]. I hope this paper has given you insight into the many applications Green's functions have towards quantum mechanics. I also hope that this paper has given you the curiousity to further pursue the topic and see the even larger breadth of applications Green's functions have in the field of mathematical physics. In either case, I hope you enjoyed this very brief introduction to Green's functions and quantum mechanics, and I thank you for taking the time to read this paper!

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