Math 336 Sample Problems

One notebook sized page of notes will be allowed on the test. The test will cover up to §4.4 in the text (excluding those sections for which there was no homework).

1. p. 195, # 7

2. p. 195, # 12

3. p. 253, # 15

4. p. 253, # 16

5. Is there an analytic function f such that $|f(z)| < 1$ for $|z| < 1$, with $f(0) = \dfrac{1}{2}$ and $f'(0) = 1$? Hint: Use Schwarz's Lemma.

6. Using Rouché's theorem, show that $z^5 + 5z^3 + z - 2$ has three roots in the set $\{z : |z| < 1\}$.

7. Let U be an open set in $\mathbf{C}$ and let f and g be analytic functions on U. Suppose $|g(x)| < |f(x)|$ for all $x \in U$. Then prove that there is an analytic function h defined on U so that $\dfrac{f+g}{f} = \exp(h)$ on U.

8. Let $u(z)$ be harmonic in all of $\mathbf{C}$. Suppose $|u(z)| \leq c|z|^n$ for some positive constant c. Prove that u is the real part of a complex polynomial of degree n.

9. There may be homework problems or example problems from the text on the midterm.