Problem 14 §6.4

Prove that for small $x$,

\[ \frac{1}{x} \log(1 + x) = 1 - \frac{x}{2} + \frac{x^2}{3} + \ldots, \]

and hence that

\[ (1 + x)^{\frac{1}{x}} = e \left( 1 - \frac{x}{2} + \frac{11}{24} x^2 + \ldots \right). \]

Hence prove that

\[ \frac{e - (1 + \frac{1}{n})^n}{1/n} \to \frac{e}{2} \text{ as } n \to \infty. \]