Theorem 1. \( \text{Let } S_k \text{ be a decreasing sequence of non-empty compact sets } (S_{k+1} \subset S_k). \text{ Then } \cap S_k \neq \emptyset. \)

Proof. Let \( x_k \in S_k \). Then \( x_k \in S_1 \) for all \( k \). Hence there is a subsequence \( x_{k_j} \) that converges to a point \( a \in S_1 \). But ultimately all points of \( \{x_{k_j} : j \geq N\} \) are in \( S_i \) for each fixed \( i \). Since \( S_i \) is compact, \( a \in S_i \).

This is true for all \( i \), so \( a \in \cap S_i \neq \emptyset. \)

Corollary 1. If \( S_j \) is a decreasing sequence of compact sets and \( \cap \cap S_j = \emptyset \) then \( S_j = \emptyset \) for some \( j \).