Math 335 Sample Problems

One notebook sized page of notes (*one side*)will be allowed on the test. You may work together on the sample problems – I encourage you to do that. The test will cover sections 4.5, 4.7, 5.6, 5.7, 5.8, 8.1, 8.2, 8.3. The midterm is on Monday, March 4.

1. Let C be the curve of intersection of $y + z = 0$ and $x^2 + y^2 = a^2$ oriented in the counterclockwise direction when viewed from a point high on the z-axis. Use Stokes' theorem to compute the value of $\int_C (xz + 1)dx + (yz + 2x)dy$.

2. (a) Prove that $\int_C \frac{-ydx + xdy}{x^2 + y^2}$ is not independent of path on $\mathbf{R}^2 - \mathbf{0}$.

   (b) Prove that $\int_C \frac{xdx + ydy}{x^2 + y^2}$ is independent of path on $\mathbf{R}^2 - \mathbf{0}$. Find a function $f(x, y)$ on $\mathbf{R}^2 - \mathbf{0}$ so that $\nabla f = (\frac{x}{x^2 + y^2}, \frac{y}{x^2 + y^2})$.

3. Folland §7.5, #9.

4. Folland, §7.5, #13.

5. Folland, §7.5, #14.

6. Prove that

$$\int_0^1 (1 - t^4)^{-1/2} dt = \frac{\Gamma(\frac{5}{4})\sqrt{\pi}}{\Gamma(\frac{3}{4})}.$$

7. Let f be a 2π-periodic function and let a be a fixed real number and let a new function g be defined by $g(x) = f(x - a)$. What is the relation between the Fourier coefficients $\widehat{f}(n)$ and $\widehat{g}(n)$?

8. Find the Fourier series of

$$\frac{1 - r^2}{1 - 2r\cos x + r^2}$$

   where $0 \leq r < 1$. (You don't need to integrate.)

9. Let f be a 2π-periodic, piecewise smooth function. Let $\widehat{f}(n)$ be the complex Fourier coefficients of f. Show that there is a constant M (which will depend on f) such that $|\widehat{f}(n)| < M/|n|$ for all $n \neq 0$. Do **not** assume f is continuous.

10. Suppose f_k is a sequence of Riemann integrable functions on $[0, 2\pi]$ such that $\lim_{k\to\infty} \int_0^{2\pi} |f_k - f| = 0$. Prove that the Fourier coefficients satisfy $\lim_{k\to\infty} \widehat{f_k}(n) = \widehat{f}(n)$ for each n.

11. Suppose f and g are 2π-periodic and Riemann integrable on compact subsets of $\mathbf{R}$. Suppose also that $f(x) = g(x)$ in a neighborhood of a point x_0. Suppose that the Fourier series for one of the functions converges at x_0. Prove that the other series converges and

$$\sum_{-\infty}^{\infty} \widehat{f}(n)e^{inx_0} = \sum_{-\infty}^{\infty} \widehat{g}(n)e^{inx_0}.$$

12. There may be homework problems or example problems from the text on the midterm.