George Green, Mathematician and Physicist 1793-1841
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George Green, mathematician and physicist 1793 – 1841

D.M. CANNELL and N.J. LORD

These past two years have seen the bicentenaries of Michael Faraday, Charles Babbage and John Frederick Herschel. A fourth contemporary, who deserves to rank with these, is George Green, the bicentenary of whose birth will be marked by the dedication of a plaque in Westminster Abbey. His memorial will be in proximity to those commemorating Newton, Kelvin, Faraday and Clerk Maxwell. Those to the Herschels (William and John) and Stokes are close by. Green's memorial designates him "Mathematician and Physicist". Most mathematicians will know of Green's theorem and Green's functions; physicists find his papers seminal to the study of, for example, solid state physics and elasticity and, since the mid-twentieth century, Green's functions have become an indispensable technique for those working in nuclear physics. It was Green who first used the term "potential" in electricity. It was green who first enunciated the principle of the conservation of energy. It was he who closely studied the conditions for reflection at an interface which provided the first convincing explanation of total internal reflection, the basis of fibre optics and thus an important element in telecommunications. Many developments in modern technology, semiconductors and superconductors, geological sounding and seismology, medical electronic scanning, owe a debt to Green.

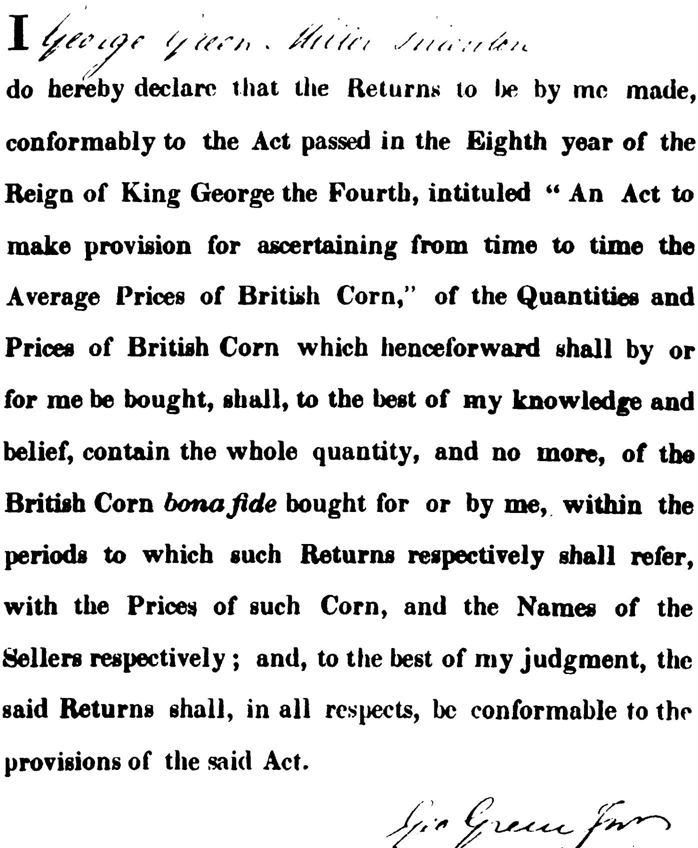
So why does not such a man enjoy a reputation equal to those of his illustrious contemporaries? One reason perhaps lies in the events of his personal life: his irregular family circumstances deprived him of standing and heritage in the town of his birth. Until his later years, he worked alone and unknown, and his early death at forty-seven allowed him only eleven years of intellectual activity and the publication of a mere ten papers. A second reason was that Green's work was much in advance of contemporary thought. As Einstein commented when looking through a copy of Green's *Essay* of 1828 during a visit to Nottingham in 1930, Green was twenty years ahead of his time. Green's reputation is almost entirely posthumous and but for a fortunate incident most of his valuable work might have been overlooked for good. Thirdly, as a theorist, Green did not produce spectacular and practical discoveries like those of Faraday, nor did he possess the experimental and entrepreneurial skills of Kelvin. Despite Kelvin's admiration and sponsorship of Green and his adoption of his work in electricity and magnetism as a basis for his own research, George Green might have remained a relatively obscure nineteenth century figure, were it not for the realisation of the aptness of his work to modern science and in particular, since the 1950s, to quantum mechanics. A final reason militating against Green's name becoming widely known to the general public is the advanced nature of his mathematics. This fact powerfully indicates the full stature of Green, a self-taught genius, but has resulted in ignorance and lack of recognition of his true worth. The sophisticated nature of Green's techniques and the fields of research where they are applied suggest that Green's name will never become as well known as Babbage's or Faraday's. The appreciation of Green that follows will be in two parts: the first looking at Green the man, the second at Green's published work.

George Green was born on 14th July 1793, the only son of a Nottingham baker. His cousin and brother-in-law, William Tomlin, writing after the mathematician's death, tells us that at the age of eight young George displayed a lively interest in 'the mathematics', such that his father sent him to the leading academy of the town. The proprietor, Robert Goodacre, was a young schoolmaster with quite advanced ideas on education and a large collection of 'philosophical instruments', which included for example an 'electrical machine' and an orrery. Goodacre later developed a career as a lecturer in popular science, travelling in Britain and the United States. His textbooks in arithmetic were still being reprinted some years after his death. Goodacre's Academy was, it would appear, an unusual and stimulating environment for young minds but after four terms George Green left in the midsummer of 1801, having learnt all the mathematics his masters could teach him, and went to work in his father's bakery. The business prospered and in 1807 Green Senior built a brick windmill in Sneinton, a village just outside the town boundary. Some ten years later he built a family house alongside where he, his wife and his son George went to live. His daughter Ann married her prosperous cousin, William Tomlin, in 1817 and they established themselves in one of the more prestigious parts of Nottingham.

In 1828, at the age of thirty-five, George Green published *An Essay on the Application of Mathematical Analysis to the Theories of Electricity and Magnetism*. Nothing is known of his intellectual development since he left Goodacre's Academy twenty-seven years previously. We do know that he worked long and arduous hours at his father's mill and William Tomlin tells us he found the miller's duties 'irksome'. He was known for spending any free time he had on mathematics: indeed, as legend has it, he is supposed to have used the top floor of the mill as a study. But how do we explain the appearance of a work which on the centenary of its publication was described as 'the most important intellectual event in the history of Nottingham'?

The 'Mathematical Analysis' in the *Essay* title gives pause for thought, since it was the term applied to the mathematics used on the

Continent based on the calculus as formulated by Leibniz, in contradistinction to the 'fluxions' of Newton: in Babbage's memorable phrase "*d*-ism as opposed to *dot*-age".



I George Green. Miller Sneinton

do hereby declare that the Returns to be by me made, conformably to the Act passed in the Eighth year of the Reign of King George the Fourth, intituled " An Act to make provision for ascertaining from time to time the Average Prices of British Corn," of the Quantities and Prices of British Corn which henceforward shall by or for me be bought, shall, to the best of my knowledge and belief, contain the whole quantity, and no more, of the British Corn *bona fide* bought for or by me, within the periods to which such Returns respectively shall refer, with the Prices of such Corn, and the Names of the Sellers respectively; and, to the best of my judgment, the said Returns shall, in all respects, be conformable to the provisions of the said Act.

Geo Green Jun

FIGURE 1. A miller's declaration signed by Green in 1827, the year in which his *Essay* went to press. Acknowledgment: Nottinghamshire County Records Office.

Green's knowledge of the works of Laplace, Legendre, Lacroix and others probably stems from a study of a translation of the first book of the *Mécanique Céleste* of Laplace, published in Nottingham in 1814, and which included reference to these mathematicians in the Preface. The author was John Toplis, Fellow of Queens' College, Cambridge. Toplis was a keen protagonist for Leibnizian mathematics, to the point of publishing his translation at his own expense. He was headmaster of the Nottingham Free Grammar School from 1806 to 1819, during George Green's adolescence and early manhood, and it is highly likely that he was Green's mentor in the early stages of his mathematical development. When Toplis returned to Queens' in 1819 Green had some seven years in which to develop further his mathematical thought. This produced amongst other things the well-known Green's functions and Green's Theorem which are found in the *Essay*.

AN ESSAY

ON THE

APPLICATION

OF

MATHEMATICAL ANALYSIS TO THE THEORIES OF ELECTRICITY AND MAGNETISM.

BY

GEORGE GREEN.

Nottingham:

PRINTED FOR THE AUTHOR, BY T. WHEELHOUSE.

SOLD BY HAMILTON, ADAMS & Co. 33, PATERNOSTER ROW; LONGMAN & Co.; AND W. JOY, LONDON; J. DEIGHTON, CAMBRIDGE;

AND S. BENNETT, H. BARNETT, AND W. DEARDEN, NOTTINGHAM.

1828.

FIGURE 2. The title page of Green's *Essay*. Acknowledgement: Nottingham University Manuscripts Department.

The *Essay* of 1828 applied mathematical analysis to the current theories of electricity and magnetism. Green is recognised as one of the first exponents of mathematical physics in England, and in all of his papers he applied mathematics to the study and measurement of physical phenomena: electricity and magnetism, the behaviour of waves in fluids and then sound and light. The choice of electricity and magnetism was another unusual feature of the *Essay*, since these subjects attracted little notice at that time in England: it was not till William Thomson (later Lord Kelvin) started work on electromagnetism in the 1840s and Faraday's more practical work became known that these topics attracted general attention. Green however was conversant with Poisson's memoirs on magnetism, published by the Institut de France from 1811 onwards and refers to them frequently in his *Essay*. Green appears to have known them in detail and not merely through abstracts in scientific journals.

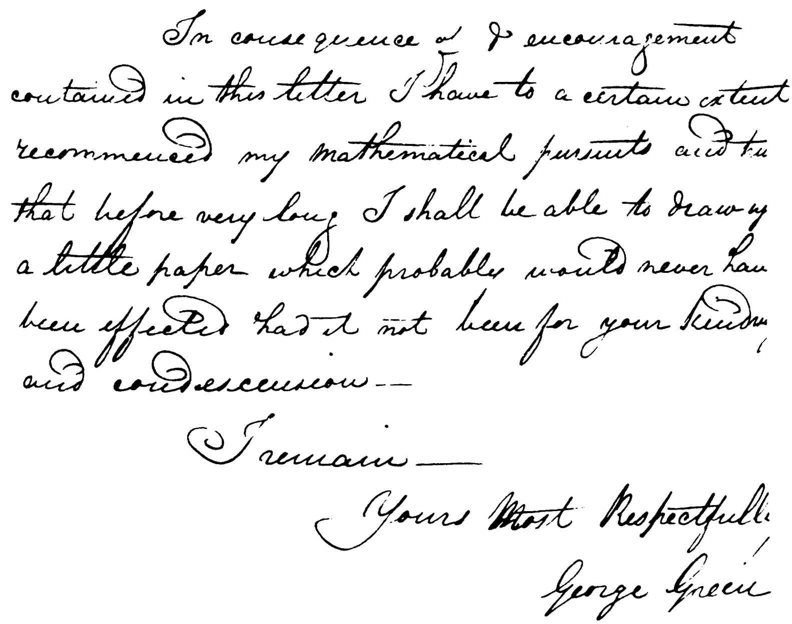
Green would have had access to these through his membership of the Nottingham Subscription Library. This was founded in 1816 by a group of professional people, a few of the local gentry, and the better-off townspeople and owners of business concerns. It was set up in Bromley House in Nottingham's Market Square and soon became a social and cultural centre. It was also a focus for the popular scientific interest of the period, as indicated in the success of Robert Goodacre, who gave a series of lectures in Nottingham in 1826. It has been assumed that the Library provided Green with study books, but apart from housing the *Transactions* of the Royal Society, to which Green makes reference, the science books acquired by the Library up to 1828 would have been too elementary to have helped Green in his reading, and it appears evident that the seminal qualities of the *Essay*, both in its mathematical and physical concepts, flow from Green's own genius. Bromley House was useful in one respect however. When Green decided to publish at his own expense and opened a subscription list, fifty-one members bought a copy, which cost seven shillings and sixpence, roughly the equivalent of a working man's wage at the time. This would indicate that they were not unaware of one of unusual talent in their midst. But in view of the following letter written in April 1828 at the time of publication, it would seem that Green, now thirty-four, and after five years' membership of the Library, had hardly established himself as one of its leading figures:

"I learn from Nottingham that Mr. G. Green is the son of a Miller, who has had only a common education in the Town, but has been ever since his mind could appreciate the value of learning immoderately fond of mathematical pursuits, and which attainments have been acquired wholly by his own perseverance unassisted by any tutor or preceptor: he is now only 26 or 27 years of age of rather reserved habits attends the business of the Mill, but yet finds time for his favourite Mathematical reading." (Green was actually thirty-four years old!)

The letter was written to Sir Edward Ffrench Bromhead, of Thurlby

Hall, Lincolnshire, who had subscribed to the *Essay* and was seeking information about the author. Bromhead had added his name to the subscription list "as Country Gentlemen often do by way of encouraging every attempt at provincial literature', as he wrote in 1845, four years after the mathematician's death. Bromhead must have been extremely surprised to read what this provincial writer had produced. The story of Green's association with Bromhead is a fascinating one but too detailed to be entered into here. Bromhead encouraged the composition and sponsored both the publication of Green's next three papers (two in the *Transactions* of the Cambridge Philosophical Society, the other in the *Transactions* of the Royal Society of Edinburgh) and later his entry into his own college of Gonville and Caius in Cambridge.

The publication of Green's *Essay* invoked no response apart from Bromhead, but although the *Essay* remained neglected until after Green's death it was Green's great good fortune that Bromhead had been a subscriber. Bromhead was not capable of recognising the full scope of Green's genius, nor the full significance of the *Essay*: that would be left to William Thomson later in the century. But he was well suited to appreciate the mathematical precocity of the work, and to be surprised at


In consequence of the encouragement contained in this letter I have to a certain extent recommenced my mathematical pursuits and trust that before very long I shall be able to draw up a little paper which probably would never have been effected had it not been for your kindness and condescension—

I remain—

Yours Most Respectfully

George Green

FIGURE 3. The final part of a letter from Green to Bromhead dated February 13, 1830, in which Green acknowledges Bromhead's help and encouragement. The "little paper" referred to is *Mathematical Investigations concerning the Laws of Equilibrium of Fluids analogous to the Electrical Fluid, with similar Researches.*

Acknowledgement: Mr and Mrs Robin German.

the use of continental analysis by an obscure working miller living in a small county town. Edward Bromhead had attended Caius College and after taking his degree had left in 1812 to spend a year studying law in the Inner Temple in London. He then returned to Lincolnshire where as the eldest son of Sir Gonville Bromhead he shortly succeeded his father as second baronet and undertook a life-time's service as High Steward of Lincoln, county magistrate and leader in the county's cultural and political affairs. Bromhead however was more than a county landowner and country gentleman. He was an outstanding mathematician in his own right. Had he not suffered from ill-health, which dogged him all his life, he might have taken the Mathematical Tripos with high honours and been recognised as one of the outstanding mathematicians of his generation. As an undergraduate at Cambridge he led the group which became the Analytical Society. Founder members included friends such as John Herschel, George Peacock and, in particular, Charles Babbage with whom Bromhead maintained a close and life-long friendship. Babbage, Herschel and Peacock, with Bromhead's active encouragement, published in 1816 a translation of the *Calcul Intégral et Différentiel* of Lacroix. In Bromhead's view the Society was influential in the setting-up of the Cambridge Philosophical Society in 1820. It published its first *Transactions* the following year. Actively promoted by Peacock and William Whewell, now established as tutors in the University, mathematical analysis swiftly replaced Newton's fluxions.

It is not surprising therefore that Bromhead sent the papers that Green wrote after the *Essay* to Cambridge for publication in the *Transactions* of the Cambridge Philosophical Society, though as a Fellow he could have sent them to the Royal Society of London – Bromhead had sent them a paper of his own which had been published in the *Transactions* of 1816. But the reputation of the Royal Society (as a progressive organisation) did not stand very high at this period and Bromhead judged that Green's papers stood a better chance of recognition in Cambridge. The first paper, sent in May 1832, was entitled *Mathematical Investigations concerning the Laws of the Equilibrium of Fluids analogous to the Electric Fluid, with other similar Researches*.

Green, now in full spate of activity, soon produced a second paper which Bromhead also forwarded to Cambridge and which was also published in the C.P.S. *Transactions*, though not till 1835. This was *On the Determination of the Exterior and Interior Attractions of Ellipsoids of Varying Densities*.

William Whewell, to whom Bromhead sent Green's papers, was a prominent member of the Cambridge Philosophical Society. He had not been one of Bromhead's inner circle but he, with Peacock, remained in the University. He established himself as a key figure in nineteenth century Cambridge, embarking on a career which would lead him to the Master's Lodge at Trinity and acquiring a reputation for activity and learning which would earn him Sydney Smith's famous dictum: "Science is his forte and omniscience his foible". In acknowledging the receipt of Green's paper on the *Attractions of Ellipsoids*, Whewell sent some advice:

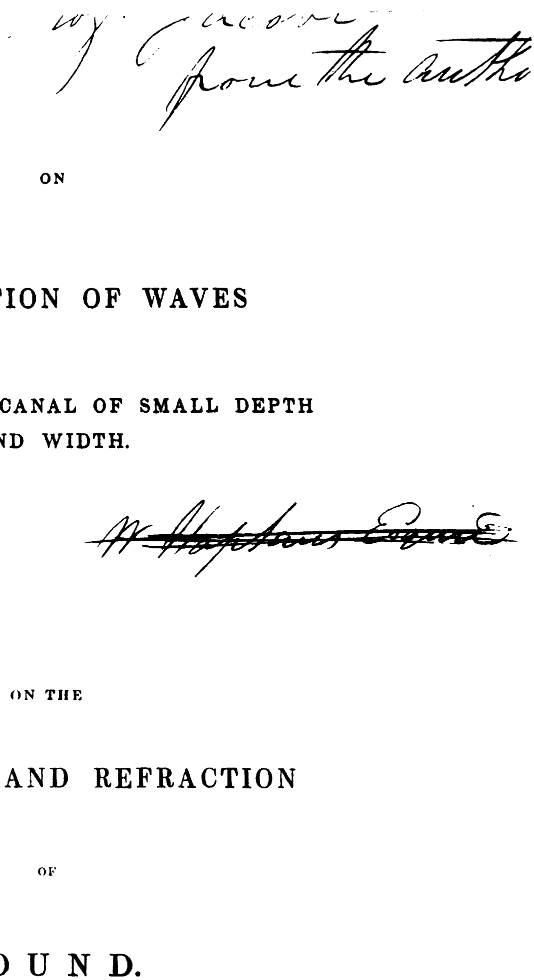
"For my own part I may observe that I read with more pleasure those analytical investigations which apply to problems which really *require* to be solved for the purposes of advancing physical science (of which questions there is no lack) than to those researches where the analytical beauty and skill are the only *obvious* merits."

The topics he suggested were the laws of heat, the motions of fluids "both with respect to the properties of waves and the laws of the tides", and "the many problems which the theory of light offers".

Green apparently took Whewell's advice. His third paper written under Bromhead's encouragement was on *Researches on the Vibration of Pendulums in Fluid Media*. Bromhead offered this paper to the Royal Society of Edinburgh, of which he was a Fellow, and it was published in the Society's *Transactions* of 1836. Green followed this with two other papers on the behaviour of water waves which were published during his time in Cambridge, *On the Motion of Waves in a Variable Canal of small Width and Depth*, published in 1838, and a *Note on the Motion of Waves in Canals*, published the following year.

In 1833, when he was forty, Green decided he would go to Cambridge. "Several kind and respected friends were anxious that he should adopt an University education several years before that circumstance actually took place", wrote William Tomlin in 1845. Green's father had died in 1829 leaving him free and with sufficient resources to sell the milling business and devote his time to mathematical study. He was now sufficiently affluent to aspire to the status of gentleman and academic, financing himself for the six years he was now to spend in Cambridge. In October 1833 he entered Bromhead's own college of Gonville and Caius and was soon recognised for his mathematical ability. "He stood head and shoulders above all others in the University", wrote a contemporary student, Harvey Goodwin, and was expected to head the final examination list as Senior Wrangler. In the event, in January 1837, he became Fourth Wrangler (the colourful algebraist James Joseph Sylvester was Second). He was now forty-five, older even than the Tutors and Fellows in the College, and the contemporary of such eminent Cambridge figures as Whewell, Peacock, Airy and the great Cambridge coach, William Hopkins.

Now a graduate with a first class degree Green stayed on at Caius hoping for a Fellowship. With examinations behind him he returned to his own researches and published six papers in the next two years. These included the two papers on waves already mentioned. Green's work on wave theory led him to investigate the reflection and refraction of sound and light.


ON

THE MOTION OF WAVES

IN A VARIABLE CANAL OF SMALL DEPTH AND WIDTH.

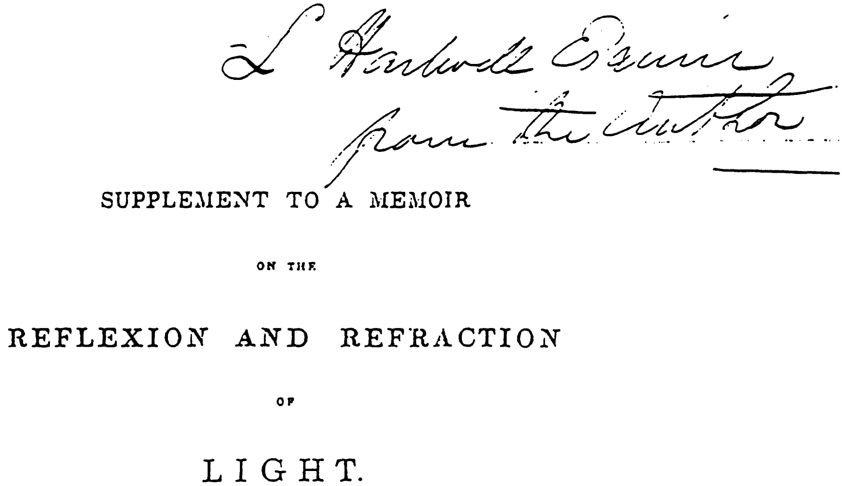
ON THE

REFLEXION AND REFRACTION

OF

SOUND.

FIGURE 4. Parts of the title pages from three of Green's later papers, which were read to the Cambridge Philosophical Society in May 1837, December 1837 and May 1839. The first (which has been trimmed) to "Professor Jacobi" raises the exciting possibility of Green's having been in contact with at least some Continental mathematicians. The second, to "W. Hopkins, Esquire" is intriguing in view of Hopkins' role in the rediscovery of Green's *Essay*. The third, below, is to "L. Hartwell, Esquire" his maternal cousin and executor, a pawnbroker in Nottingham. Acknowledgements: Nottingham University Manuscripts Department and Professor G.L. Alexanderson, Santa Clara University, California.



The paper on sound *On the Reflexion and Refraction of Sound* was published in 1838, but Green's major contribution to mathematical science after that of the *Essay* of 1828 is to be found in his three papers on light: *On the Laws of the Reflexion and Refraction of Light at the Common Surface of two non-crystallized Media, Supplement to a Memoir on the Reflexion and Refraction of Light* and *On the Propagation of Light in Crystallized Media.* These publications were his last and indicate that Green's intellectual powers in his forties were as strong as ever.

What was apparently Green's ambition – to be elected in to a Fellowship – was realised at last in October 1839. Sadly he only enjoyed the privileges and status for two terms. He returned to Nottingham in the spring of 1840, "Alas! With the opinion", wrote William Tomlin, "that he should never recover from his illness and which became verified in little more than a year's time by his decease on the 31st May 1841." The cause of death was given as influenza, but it is possible that Green's long years working as a miller had laid the seeds of a lung complaint which overtook him in middle age. He was buried with his parents in St. Stephen's churchyard in Sneinton, only a few hundred yards from the mill where he had worked for most of his life.

With Green's departure from Cambridge his reputation appears to have waned. When, four years later, in 1845, the young William Thomson rediscovered Green's *Essay* and sought information about him, Caius could do no better than refer the query to Bromhead, whose reply was the letter of 1845. This, like Tomlin's of the same date, is valuable in giving us the scant information on the mathematician vouchsafed to posterity. Bromhead's letter was cautious: "My acquaintance with the late Mr. Green was quite casual", he states at the beginning, and proceeds to mention his subscription to the *Essay*, Green's visits to Thurlby, and the latter's desire to go to Cambridge. "So much for my knowledge of poor Green", he concludes, and makes no mention of his promise or achievements. This testimony, patronising, even dismissive, is perplexing when one considers Bromhead's early enthusiasm for Green's work. Their correspondence ceased in 1834, though possibly not Green's visits to Thurlby, but the course of their relationship in the following seven years remains a mystery.

The story of Thomson's discovery of Green's *Essay* is intriguing. Anxious after his graduation to embark on the exploration of problems in electricity he came across a footnote to a memoir by Robert Murphy in which Murphy refers to "what Mr. Green, of Nottingham, in his ingenious *Essay* on the subject has denominated the Potential Function". Murphy had been the 'rapporteur' (referee) for Green's first paper sent to Cambridge by Bromhead, and a copy of the *Essay* had been sent at the same time: thus Murphy had read the *Essay*. Thomson searched in vain for a copy of Mr. Green's ingenious *Essay* in the Cambridge booksellers, but quite by accident found his coach William Hopkins had copies donated by Green. Thomson, now aged twenty-one, was departing for Paris the next day to work with French scientists during the summer. He not only became acquainted with the *Essay* en route, writing excitedly that "In 1828 [Green] had given almost all the general theorems in attraction which have since occupied Chasles, Gauss, etc.", but also showed it to the continental mathematicians whom he met, noting that, "Green's memoir creates a great sensation here. Chasles and Sturm find their own results and demonstrations in it." The German August Crelle, editor of the eponymous *Journal*, agreed to publish it and it appeared in three parts in 1850, 1851 and 1853. Thomson drew on Green's work for his own papers on electromagnetism and throughout his long life (he died in 1907 at eighty-three), ceaselessly extolled the virtues of Green. His friend and contemporary George Gabriel Stokes likewise admired Green's work and made reference in particular to his papers on hydrodynamics. Thus during the nineteenth century Green's contribution to mathematical science was recognised and Cambridge also came to recognise his importance. More recently, in [29, p153], Whittaker summarised Green's pivotal role thus: "... it is no exaggeration to describe Green as the real founder of that 'Cambridge School' of natural philosophers of which Kelvin, Stokes, Rayleigh, Clerk Maxwell, Lamb, J.J. Thomson, Larmor and Love were the most illustrious members in the latter half of the nineteenth century." It is indeed unfortunate that no portrait or even written physical description of Green survives.

If Green's reputation was slow to be established in Cambridge, the process was even slower in his native town, and it is only since the 1970s that his name has come to be known there. At the beginning of this article it was suggested that Green's lack of recognition in Nottingham was due to the circumstances of his personal life. During his years in Nottingham at least, Green appears to have been of a reserved nature, living in what was then a class-conscious age. As the son of a miller, and a working miller himself, he belonged to the trading class and only the affluence accruing from the death of his father allowed him gradually to move up the social scale. The change to gentleman and academic was only fully realised after his departure to Cambridge, when he also resigned from the Bromley House Library. In 1833 therefore he ceased to live in the town and in his will he describes himself as "late of Snenton and now of Caius College, Cambridge". Since the *Essay* was forgotten in Nottingham and his other papers had been published in Cambridge it is perhaps not surprising that he left no reputation behind him. Furthermore he had no established home in Nottingham and he gave William Tomlin's address as poste restante in vacations. Thus his only obituary, in the *Nottingham Review*, was a modest one:

"In our obituary of last week, the death of Mr. Green, a mathematician, was announced: we believe he was the son of a miller, residing near to Nottingham, but having a taste for study, he applied his gifted mind to the science of mathematics, in which he made rapid progress. In Sir Edward Ffrench Bromhead, Bart he found a warm friend, and to his influence owed much, while studying at Cambridge. Had his life been prolonged, he might have stood eminently high as a mathematician."

This was hardly the basis for the establishment of a lasting posthumous reputation and Green was totally forgotten in Nottingham. A further reason for Green's anonymity was his lack of a permanent home, since he leased the family house next to the mill when he left for Cambridge. And here we come to a totally unexpected side of Green's life which would have been unknown to Cambridge and in all probability to Bromhead. Green Senior, when he built the mill in Sneinton as an adjunct to his bakery, employed a mill manager, William Smith, who would doubtless have undertaken the training of the baker's son as a miller. By 1823, when he was thirty, George Green had formed a relationship with the miller's daughter, Jane, which resulted in the birth of a daughter. Three years later she bore him a second daughter. Both children, Jane and Mary Ann, were registered in their mother's name as illegitimate, but for the second child Jane Smith added Green as a baptismal name. At this point Jane may have recognised that her attachment to George Green was likely to be permanent but Green Senior is thought to have forbidden their marriage. Jane Smith finally bore

George Green seven children; the last, Clara, was born after his final return from Cambridge. All but the first were given the name of Green and by the time the eldest was ten she was calling herself Jane Green. The other children likewise called themselves Green and Jane Smith herself was later known as Mrs. Jane Green. The vicissitudes of the family are too detailed for full recital here, but the equivocal position of Jane Smith as a common-law wife, the illegitimate status of the children and the lack of an established family house and home all militated against the adoption of George Green as one of the worthies of Nottingham, still less as one of its most noteworthy citizens.

Green provided for his dependants. Mrs. Jane Green lived in the house Green had provided in Sneinton and died at seventy-five. John, Green's younger son, died at twenty before inheriting the mill and two daughters, Mary Ann and Catherine, died in their twenties. A third daughter, Elizabeth, married in middle life and died without issue. More is known about the remaining three children. The elder son, George, went to Goodacre's Academy, became a schoolmaster and in his late twenties enrolled at St. John's College, Cambridge, taking the Mathematical Tripos in 1859. He may possibly have taught in Cork, under the patronage of George Boole, Professor of Mathematics in the recently established Queen's College (and very probably another mathematical protégé of Edward Bromhead). In 1871 George Green committed suicide in London at the age of forty 'while in an unsound state of mind'.

Clara Green's story is also a sad one. The youngest of the seven children and unmarried, she was left to bear the full stigma of her illegitimacy in the neighbourhood where she was born and in an age which had adopted rigid standards of moral rectitude. The final inheritor of George Green's original quite considerably patrimony of 1841, she lived in penury in Sneinton and died in the workhouse infirmary in 1919 at the age of seventy-nine. Her estate – the mill, the family house and the surrounding Green's Gardens – was heavily mortgaged. The windmill had ceased working in the 1860s and was now derelict. On the assumption that Clara was the last survivor of the family the Crown acquired the property and sold it to help pay off the debt. Her grave was discovered barely three years ago. It lies utterly neglected, the headstone half sunk into the ground. The inscription records the death of Clara Green 'Daughter of the late George Green M.A. of Caius College Cambridge and formerly of Sneinton'. Clara was thirteen months old when her father died, but her testimony in her last years to inquirers from the University College in Nottingham reveals both knowledge and respect with regard to her father. Green's Fellowship is also recorded in his own epitaph on the family grave in Sneinton, some three miles distant. And therein surely lies the reason why he never married Jane

Smith and legitimised his children. Arriving in Cambridge as a married man he would not have been eligible for a Fellowship. Jane Smith, one might think, was prepared to pay the price for the realisation of George Green's ambition, but the final onus fell on Clara and possibly on her brother George too.

The wider recognition of George Green outside the restricted circle of specialist mathematicians and physicists has taken most of this century to accomplish. In 1907 Joseph Larmor in Cambridge was inquiring of Lord Kelvin the circumstances of his discovery of Green's *Essay* in 1845. At the same time members of the University College in Nottingham also began enquiries in Sneinton. They were able to meet Clara Green before she died but were given short shrift by a proud, embittered and reclusive old woman in her late seventies: "if information was wanted why was it not sought before?". The Mathematical Section of the British Association for the Advancement of Science, meeting in Nottingham in 1937, visited his grave and as a result of finding it in a rather dilapidated state sent a letter of protest to the Lord Mayor. As a result George Green's grave has since been maintained in a reasonable state by the City Council. Finally, in the 1970s a group of physicists and mathematicians from Nottingham University, under the leadership of Professor Lawrie Challis, launched a project for the restoration of the mill as a memorial to George Green. This was successfully concluded some ten years later and Green's Mill now grinds flour daily. Adjoining is the Green Science Centre where 'hands-on', 'push-button' science exhibits attract the young and not so young. The complex forms one of the most popular of Nottingham's museums, an establishment due jointly to academic initiative and to civic generosity and support.

Following local recognition comes national acclaim with the celebrations for the bicentenary of Green's birth during July 1993. On the anniversary of the actual birth date, 14th July, the University of Nottingham will welcome and confer honorary degrees on two scientists who were closely involved with the development of the use of Green's mathematics in quantum mechanics in the 1940s and 50s. Professor Julian Schwinger of the University of California, who shared the Nobel Prize for Physics in 1965 with S. Tomonaga and the late R.P. Feynman, and Professor Freeman Dyson of the Institute for Advanced Study at Princeton, will each read a paper on their work in this field. An account of their experiences is eagerly awaited by Green enthusiasts, since this important stage in the long history of Green's mathematics has remained relatively obscure. On 16th July papers will be read at the Royal Society followed by the dedication of a plaque to Green after Evensong at 5pm in Westminster Abbey. Its site in the Sanctuary adjoining Newton's ledger and near to Green's own near contemporaries, in particular Kelvin and

Faraday, establishes his true status, too long denied him. Some months before Green went up to Cambridge, Bromhead invited him to join a reunion there of some of his undergraduate friends. But Green declined the invitation: "You were kind enough to mention a journey to Cambridge on 24th June to see your friends Herschel Babbage and others who constitute the Chivalry of British Science. Being as yet only a beginner I think I have no right to go there and must defer that pleasure until I shall have become tolerably respectable as a man of science should that day ever arrive."

Green's mathematics

We turn now to a more detailed consideration of Green's papers. They were collected and prepared for publication by N.M. Ferrers in 1871 in an edition thankfully reprinted by Chelsea in 1971, [1]. What follows is a factual account of them as seen through modern eyes; for convenience the papers are treated chronologically – that is, in the order they were mentioned above – and for brevity they are referred to by the key-words in their titles. At least two fascinating pieces of detective work remain to be undertaken. First, there is the teasing question of Green's use of the sources available to him when he composed his *Essay* and on which a detailed textual critique might shed some light; second, the route by which the technique of Green's functions became assimilated into mainstream mathematical methodology remains to be charted.

The Essay

After a short preface in which Green launches his work with some diffidence: "... it is hoped the difficulty of the subject will incline mathematicians to read this work with indulgence, more particularly when they are informed that it was written by a young man, who has been obliged to obtain the little knowledge he possesses, at such intervals and by such means, as other indispensable avocations which offer but few opportunities of mental improvement, afforded." and a detailed summary, the *Essay* splits into three roughly equal parts – the first, on which we shall concentrate, contains mathematical preliminaries and the other two contain applications of these results to then contemporary concerns in electrostatics and magnetism. For convenience we shall use modern vector analysis notation: this was not widely used until the early part of this century.

The utility of the function

$$V(x_0) = \int_\tau \frac{\rho(x)\, d\tau}{|x - x_0|}$$

(the integral taken over a volume (τ) or surface (σ) in/on which there resides a mass/electricity distribution, density ρ) in studying inverse square law force fields had been recognised by mathematicians such as Lagrange, Laplace and Poisson, but Green was the first fully to appreciate that the whole theory of electrostatics, applicable to all shapes of conductors, could be based on it provided that one could solve the inverse problem – given V find ρ . He was also the first to give it a name: "we ... will ..., for abridgement, call it the *potential function*". (It is interesting that Gauss, apparently independently, employed the same name in his memoir on attraction of 1839.) It was known that $-\nabla V$ gave the force on a unit charge arising from the given distribution of electricity and Green reproduces Poisson's proof that $\nabla^2 V = -4\pi\rho$ at points where the space charge density is ρ (this subsumes Laplace's equation $\nabla^2 V = 0$ in free space), deducing that inside a closed conductor in equilibrium, since the force field, and hence ∇V, is constant, $\nabla^2 V = 0$ whence $\rho = 0$, i.e. the charge of a perfect conductor resides on its surface. If the charged conductor is placed in a (known) external electric force field, a redistribution of charge occurs on the conductor to produce an equilibrium charge density which annihilates the external field. By the argument just given, since $\nabla V = 0$ inside the conductor, V is constant inside and on the surface of the conductor. Although several of Green's applications are concerned with this situation, he considers the more general problem: how can ρ , a surface charge density, be deduced from a knowledge of V *on the surface*?

His basic tools are *Green's identities*: for suitably smooth U, V over a region of space τ bounded by a surface σ (and with normals pointing outwards from σ):

$$\int_\tau (U\nabla^2 V + \nabla U\cdot\nabla V)\, d\tau = \int_\sigma U\,\nabla V\cdot d\sigma,$$

and – interchanging U, V and subtracting:

$$\int_\tau (U\nabla^2 V - V\nabla^2 U)\, d\tau = \int_\sigma (U\nabla V - V\nabla U)\cdot d\sigma.$$

Nowadays these are usually deduced by applying the divergence theorem to $U\nabla V$; Green gives a direct proof using integration by parts on $\int \nabla U.\nabla V\, d\tau$. (Green's name is sometimes attached to the divergence theorem which may, indeed, be proved by a very similar argument to Green's – compare [1, pp24-6] and [19, p3] – but any such attribution is fraught with difficulty: the theorem was 'in the air' in the early nineteenth century and Gauss, Poisson, Ostrogradsky, ... all have equally strong claims. Similar comments apply, with even greater force, to the so-called Green's theorem in the plane!)

The key to using Green's identities lies in the freedom over the choice of U, V, and of τ, σ, enhanced by Green's brilliant realisation that

(3.) Before proceeding to make known some relations which exist between the density of the electric fluid at the surfaces of bodies, and the corresponding values of the potential functions within and without those surfaces, the electric fluid being confined to them alone, we shall in the first place, lay down a general theorem which will afterwards be very useful to us. This theorem may be thus enunciated:

Let U and V be two continuous functions of the rectangular co-ordinates x, y, z, whose differential co-efficients do not become infinite at any point within a solid body of any form whatever; then will

$$\int dxdydz\, U\delta V + \int d\sigma\, U\left(\frac{dV}{dw}\right) = \int dxdydz\, V\delta U + \int d\sigma\, V\left(\frac{dU}{dw}\right);$$

the triple integrals extending over the whole interior of the body, and those relative to $d\sigma$, over its surface, of which $d\sigma$ represents an element: dw being an infinitely small line perpendicular to the surface, and measured from this surface towards the interior of the body.

To prove this let us consider the triple integral

$$\int dxdydz \left\{ \left(\frac{dV}{dx}\right)\left(\frac{dU}{dx}\right) + \left(\frac{dV}{dy}\right)\left(\frac{dU}{dy}\right) + \left(\frac{dV}{dz}\right)\left(\frac{dU}{dz}\right) \right\}.$$

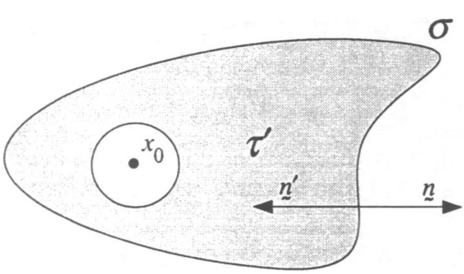
The method of integration by parts, reduces this to

$$\int dydz\, V''\frac{dU''}{dx} - \int dydz\, V'\frac{dU'}{dx} + \int dxdz\, V''\frac{dU''}{dy} - \int dxdz\, V'\frac{dU'}{dy}$$
$$+ \int dxdy\, V''\frac{dU''}{dz} - \int dxdy\, V'\frac{dU'}{dz} - \int dxdydz\, V\left\{\frac{d^2U}{dx^2} + \frac{d^2U}{dy^2} + \frac{d^2U}{dz^2}\right\};$$

the accents over the quantities indicating, as usual, the values of those quantities at the limits of the integral, which in the present case are on the surface of the body, over whose interior the triple integrals are supposed to extend.

FIGURE 5. Green's original statement of "Green's theorem", from his *Essay* of 1828. Note that, in accordance with the conventions of his day, he uses "d" for all types of derivatives, a single integral sign for multiple integrals, and he writes δ for ∇^2. He also uses inward-drawn normals. Acknowledgement: Nottingham University Manuscripts Department.

valuable information could be extracted from certain "*singular values* of a given function": as Littlewood once felicitously remarked, Green made an infinity of a function do positive work instead of being a disaster. Thus he points out that if U(x) behaves like $1/|x - x_0|$ near x_0 inside τ then, applying his second identity to the 'exclusion zone' τ' outside a small sphere radius ε centred on x_0, gives



$$\int_{\tau'} (U\nabla^2 V - V\nabla^2 U)\, d\tau = \int_{\sigma} (U\nabla V - V\nabla U).\, d\sigma + \underbrace{\int_{|x-x_0|=\varepsilon} U\nabla V.\, d\sigma}_{O\left(\frac{1}{\varepsilon}.1.\varepsilon^2\right)} - \underbrace{\int_{|x-x_0|=\varepsilon} V\nabla U.\, d\sigma}_{\approx V(x_0).\,\frac{\varepsilon^2}{\varepsilon^3}.\varepsilon.4\pi}$$

which, on letting $\varepsilon \to 0$, gives an additional term $-4\pi V(x_0)$ on the right-hand side.

Fixing V as the potential function for a charge density ρ on the surface and first choosing $U(x) = 1/|x - x_0|$, both of which satisfy Laplace's equation away from x_0 and σ, he obtains immediately (but in passing) the representation theorem:

$$V(x_0) = \frac{1}{4\pi}\int_{\sigma} \frac{1}{|x-x_0|}\frac{\partial V}{\partial n} - V\frac{\partial}{\partial n}\left(\frac{1}{|x-x_0|}\right) d\sigma$$

which together with

$$0 = \int_{\sigma} \frac{1}{|x-x_0|}\frac{\partial V}{\partial n'} - V\frac{\partial}{\partial n'}\left(\frac{1}{|x-x_0|}\right) d\sigma\ ,$$

obtained by applying the identity to the region between τ and a large sphere radius R, and letting $R \to \infty$, yields:

$$V(x_0) = \int_{\sigma} \frac{1}{4\pi}\frac{1}{|x-x_0|}\left(\frac{\partial V}{\partial n} + \frac{\partial V}{\partial n'}\right) d\sigma\ .$$

Comparing this with the definition of $V(x_0)$ and with a physicist's faith in the uniqueness of ρ, we must have that ρ is given by

$$\rho = \frac{1}{4\pi}\left(\frac{\partial V}{\partial n} + \frac{\partial V}{\partial n'}\right)\ .$$

Thus, if V can be found away from σ, ρ can be found immediately by differentiation. Green underlines the significance of the discontinuity in the normal derivatives as σ is traversed by carefully using separate notations V, $\overline{V}$ and V′ for the potential at points respectively inside, on and outside σ.

For his second choice of U: "conceive the surface to be a perfect conductor put in communication with the earth, and a unit of positive electricity to be concentrated in the point x_0, then the potential function arising from x_0 and from the electricity it will induce upon the surface, will be the required value of U." Writing $U = G(x, x_0)$ for clarity, we have $G = 0$ on σ. $\overline{\nabla}^2 G = 0$ apart from at x_0, and $G(x,x_0) \approx 1/|x - x_0|$

near x_0 whence

$$0 = \int_\sigma (0 - V\nabla G).d\sigma - 4\pi V(x_0) \quad \text{or} \quad V(x_0) = -\frac{1}{4\pi}\int_\sigma V\frac{\partial G}{\partial n}\, d\sigma .$$

(Points outside τ may be handled similarly, provided we insist that $V(x_0) \to 0$ as $|x_0| \to \infty$.) Green further shows that G satisfies the reciprocity relation $G(x, x_0) = G(x_0, x)$ and, later in the *Essay*, derives the Green's function for a sphere. It is also most striking that he checks " ... that *whatever* the value of V [on the surface] may be, the general value of V deduced from it by the formula just given shall satisfy the equation $\nabla^2 V = 0$ ", recognising the need to check continuity up to the boundary.

This famous representation theorem, giving explicitly the solution of $\nabla^2 V = 0$ in a region in terms of the value of V on the boundary, and in terms of the *Green's function* of the region (so named by Riemann) which depends solely on the geometry of the region, not on V, completes Green's mathematical preliminaries. The method, that of synthesising (by integration) a system's response to a given disturbance in terms of its response to a standard input (in this case a unit of charge at x_0) has in this century become an adaptable and illuminating rationale for scientists tackling a vast range of problems in both classical and modern physics as well as opening a veritable can of worms for pure mathematicians centred on the legitimacy of what are essentially Dirac delta-function methods – issues that were not fully resolved until Schwartz's theory of distributions in the 1950s.

Fluids

In this paper, Green investigates the equilibrium density produced by a hypothetical fluid the constituent particles of which repel each other according to an inverse nth power law. Although he feared that taking $n \neq 2$ might lead readers to "think my paper rather too hypothetical", he thought the general case "more likely to throw light on the intimate constitution of natural bodies than many others". Much of the text is concerned with cases in which there is circular or spherical symmetry and, in a display of analytical pyrotechnics, he freely adapts and shows a complete mastery of the methods of harmonic analysis evolved by Laplace and Legendre to deal with these.

Ellipsoids

The knotty problem of determining the gravitational attraction of ellipsoids had been a popular one with British mathematicians since Newton and Maclaurin and Green describes an original, self-contained and technically ingenious approach to the integrations involved which is able simultaneously to deal with interior and exterior points. This paper is notable for his willingness to consider a general $1/r^n$ potential and an ellipsoid of variable density in $\mathbf{R}^s$ (even his commentator Ferrers was dubious about this, writing: "It is of course possible to adapt the formula of this paper to the case of nature by supposing $s = 3$.") and for a clear understanding of what later (after Riemann) became known as Dirichlet's principle; specialising Green slightly, in $\mathbf{R}^3$ this asserts that, among all functions V having prescribed values on a surface σ bounding a region τ, the one minimising

$$\int_\tau |\nabla V|^2 \, d\tau$$

also satisfies Laplace's equation in τ. Taking the existence of some minimising function, V_0, for granted – a cause of much anxiety to later mathematicians – Green gives a standard calculus of variations argument to show that $\nabla^2 V_0 = 0$ and deduces uniqueness (his real goal) by observing that if V_1 is another minimising function (so that $\nabla^2 V_1 = 0$ as well) then the *same* variational argument shows that

$$\int_\tau |\nabla(V_0 + a(V_1 - V_0)|^2 \, d\tau$$

is *independent* of a, which can only happen if $V_1 = V_0$. Green was pleased with this paper, writing to Bromhead: "I flatter myself that the simplicity of the results is such as to place in a clear point of view the great advantage of the modern methods ...".

Pendulums

In this short paper, Green calculates the correction to the time of (small) swings of an ellipsoidal pendulum bob moving in a fluid, showing it to arise from a term due to the buoyancy of the fluid and a term due to the action of the fluid flowing over the bob. Here, as in his later papers, all fluids are inviscid and incompressible and all motions irrotational. Green feared that "Bessel may have anticipated me" and dismissed his "hastily written" paper as "rather an unimportant one" but later commentators have identified his solution, albeit to a very specific problem, as something of a landmark in fluid mechanics.

The four "pre-Cambridge" works occupy almost three-quarters of Green's published writings. His later papers all deal with waves of various types and although both the topics for investigation and the crisper, more clinical style in which they are written were influenced by his contacts with Bromhead and the Cambridge mathematicians, he does to some extent foreshadow them in his Preface to the *Essay*: " . . . the theory that supposes light to depend on the undulations of a luminiferous fluid . . . may furnish a useful subject of research, by affording new opportunities of applying the general theory of the motion of fluids . . . . although we have long been in possession of the general equations on which this kind of motion depends, we are not yet well acquainted with the various limitations it will be necessary to introduce, in order to adapt them to the different physical circumstances which may occur."

The two papers on waves in canals

Take the plane (xz) perpendicular to the ridge of one of the waves supposed to extend indefinitely in the direction of the axis y, and let the velocities of the fluid particles be independent of the co-ordinate y. Then if we conceive the axis z to be directed vertically downwards, and the plane (xy) to coincide with the surface of the sea in equilibrium, we have generally,

$$gz - \frac{p}{\rho} = \frac{d\phi}{dt},$$

$$0 = \frac{d^2\phi}{dx^2} + \frac{d^2\phi}{dz^2}.$$

The condition due to the upper surface, found as before, is

$$0 = g\frac{d\phi}{dz} - \frac{d^2\phi}{dt^2}.$$

From what precedes, it will be clear that we have now only to satisfy the second of the general equations in conjunction with the condition just given. This may be effected most conveniently by taking

$$\phi = H\epsilon^{-\frac{2\pi}{\lambda}z} \sin\frac{2\pi}{\lambda}(v't - x),$$

by which the general equation is immediately satisfied, and the condition due to the surface gives

$$g = \frac{2\pi}{\lambda}v'^2, \quad \text{or } v' = \sqrt{\frac{g\lambda}{2\pi}},$$

where λ is evidently the length of a wave. Hence, the velocity of these waves vary as $\sqrt{\lambda}$, agreeably to what Newton asserts. But the velocity assigned by the correct theory exceeds Newton's value in the ratio $\sqrt{\pi}$ to $\sqrt{2}$, or of 5 to 4 nearly.

FIGURE 6. Green's analysis of the motion of deep sea water waves, in which he corrects an error of Newton's. (The waves are assumed small, so a second-order term $\frac{1}{2}|\nabla\phi|^2$ has been omitted from the first equation.)

Acknowledgement: Nottingham University Manuscripts Department.

The initial paper treats the motion of 'small' water waves in a thin, shallow rectangular canal of width β and depth γ. He obtains the relevant equation governing ϕ_0, the leading term of the disturbance, by a standard 'expand and ignore the second order terms' approach, but then allows that β, γ "be functions of x [the distance along the canal] which vary slowly [compared with the wave], so that . . . $\beta = \psi(wx)$ where w is a very small quantity." This time ignoring w^2 terms, in an approach which anticipates the WKB method by almost a century, he is able to complete his analysis showing that the height of the wave depends on $\beta^{-1/2}\ \gamma^{-1/4}$, on its length λ, on $\gamma^{1/2}$ and that its speed is $\sqrt{(g\gamma)}$. He is spurred to a follow-up note on reading of "cases of propagation of what Mr Russell denominates the 'Great Primary Wave'", the earliest report of experimental work on what are now called solitons. He derives the modifications of the wave speed formula to $\sqrt{(g\gamma/2)}$ for a triangular canal and to $\sqrt{(g\lambda/2\pi)}$ for deep sea waves; just as Laplace's adiabatic argument had corrected Newton's isothermal formula for the speed of sound in air, so the latter formula corrects Newton's $\sqrt{(g\lambda)}/\pi$, for the speed of water waves. Green also points out for the first time that the particles in a deep sea wave execute circular motion.

Sound

Here, Green investigates the phenomena that occur when waves travelling through one fluid encounter a different fluid, for example, sound waves passing from air to water. In order to expose an inadvertent, but manifest, error in Poisson's work, he deliberately concentrates on a special case: a plane wave and two media separated by an infinite plane. Simply by analysing the wave equation in fluids together with mathematically appropriate boundary conditions at the interface, he is able faithfully to reproduce the laws of reflection and refraction, deduce the intensities of reflected and refracted rays and to explain fully the phenomenon of total internal reflection with, for elastic media, the accompanying change of phase which occurs. He notes that, in the latter case, his formulae tantalisingly agree with experimental observations on light polarised perpendicular to the plane of reflection.

The three papers on light

Light posed one of the most pressing problems to the physicists of the time. The experimental work of Young, Fresnel and other led to the supposition that light consisted of a transverse wave motion and the luminiferous ether was posited as that elastic (necessarily) solid medium which transmitted these vibrations: Maxwell's electromagnetic theory was still twenty years away! In the end, all attempts to model the ether as a conventional substance had problems in reconciling theory to observation – Green's was no exception and thus controversial – but his revolutionary approach to obtaining the equations governing the vibrations of an elastic solid (remembered today in the 'Cauchy-Green tensor') has earned him a prominent place in the history of elasticity: in Love's estimation [18], "The revolution which Green effected . . . is comparable in importance with that produced by Navier's discovery of the general equations."

Let us conceive a mass composed of an immense number of molecules acting on each other by any kind of molecular forces, but which are sensible only at insensible distances, and let moreover the whole system be quite free from all extraneous action of every kind. Then x y and z being the co-ordinates of any particle of the medium under consideration when in equilibrium, and

$$x + u, \qquad y + v, \qquad z + w,$$

the co-ordinates of the same particle in a state of motion (where u, v, and w are very small functions of the original co-ordinates (x, y, z) of any particle and of the time (t)), we get, by combining D'Alembert's principle with that of virtual velocities,

$$\Sigma Dm \left\{ \frac{d^2 u}{dt^2} \delta u + \frac{d^2 v}{dt^2} \delta v + \frac{d^2 w}{dt^2} \delta w \right\} = \Sigma Dv \,.\, \delta\phi \quad (1);$$

Dm and Dv being exceedingly small corresponding elements of the mass and volume of the medium, but which nevertheless contain a very great number of molecules, and $\delta\phi$ the exact differential of some function and entirely due to the internal actions of the particles of the medium on each other. Indeed, if $\delta\phi$ were not an exact differential, a perpetual motion would be possible, and we have every reason to think, that the forces in nature are so disposed as to render this a natural impossibility.

FIGURE 7. Green's use, for the very first time, of a principle equivalent to the conservation of energy. The statement about perpetual motion seems to indicate a deep understanding of the properties of conservative fields.

It is convenient to consider his three papers on light together. Rather than assume some ad hoc physical hypothesis about the structure of the ether, he proceeds more abstractly:

"The principle selected as the basis of the reasoning contained in the following paper is this: In whatever way the elements of any material system may act upon each other, if all the internal forces exerted be multiplied by the elements of their respective distances, the total sum for any assigned portion of the mass will always be the exact differential of some function." Although he does not phrase this in energy terms, this amounts to assuming the existence of a potential energy function ϕ for the 'elastic energy' stored in the medium for which the conservation of energy holds – the first time that this principle had been used. It is hard to gauge precisely how Green viewed this principle but he does seem to have regarded it as more than a mere mathematical convenience; rather tantalisingly, the desirable *mathematical* property that ϕ be an exact differential coincides with the *physically* desirable property that ϕ be a conservative field!

He then assumes that ϕ can be expanded as a power series in the six linear and angular strain components and observes that, since at equilibrium the degree one terms vanish, the leading terms are those twenty-one terms of (total) degree two. All twenty-one may be needed for a 'crystallized' (= non-isotropic) medium, but any assumptions about spatial symmetry lead to a reduction, as far as two for a non-crystallized (= totally isotropic) medium. He does not stop to dwell on their physical meaning, but proceeds to analyse refraction and reflection phenomena in a similar manner to that he employed for sound; unfortunately though, his conclusions do not match with observation as well as they might have done.

In any attempt to evaluate the corpus of Green's published work, the *Essay* stands alone. It is a work of striking originality and he hit a rich vein. The later papers are arguably more mainstream and reflect more closely the specific concerns of the age, but Green's hallmark is clear on them as well – a reluctance to make unwarranted physical assumptions (preferring instead more general mathematical hypotheses) and an eye for the incisive special case. Thus one of his champions, Stokes, enthused:

"Indeed Mr Green's memoirs are very remarkable, both for the elegance and rigour of the analysis, and for the ease with which he arrives at most important results. This arises in a great measure from his divesting the problems he considers of all unnecessary generality: where generality is really of importance, he does not shrink from it."

and a century later, Whittaker [29, p153] reflected enticingly:

"Green undoubtedly received his own early inspiration from [the great French analysts] . . . but in the clearness of insight and conciseness of exposition he far excelled his masters, and the slight volume of his collected papers has to this day a charm wanting in their more voluminous writings."

Notes on the references and on Green's Mill.

When [3] is published in May this year, it will be by far the most detailed account yet written of Green's life. Until then [4], [5], [6], [7], [8], [9] and [10] all contain mixtures, in various proportions, of biographical and mathematical details: an invaluable catalogue of primary sources is [2]. Kelvin's admiration for Green's work shines through his early papers, see [26, Arts I–XV]. Green's central place in the early history of potential theory was cemented by Thomson and Tait's influential text [25]. Ramsey's [19] is a tighter account along similar lines whilst Kellogg in his classic book [17] squarely addresses the rather fussy mathematical technicalities involved in a rigorous mathematical treatment; both also discuss the work on attractions of ellipsoids which Green generalised. Wider historical perspectives are provided by [14] and [24]. Green's sequence of deductions (from his identities to his solution of the Dirichlet problem) remains to this day a popular one in textbooks. A recent and typical example is [23]. The prominence of Green's functions as a unifying tool for efficiently solving problems in a wide variety of physical settings and in an insightful way is exemplified by books such as [12] and [20]: it is interesting to note that, even in the late 1960s, Roach lamented in the latter that, "... it is only comparatively recently that [Green's functions] have begun to emerge from the realms of more formal and abstract analysis as a potential everyday tool for the practical study of boundary value problems."

The nature of the controversy surrounding Green's work in elasticity is described in [27] and [29] and Whittaker also explores the significance of Green's work in the context of nineteenth century theories of the ether, commenting that, "Green, though inferior to Cauchy as an analyst, was his superior in physical insight." Other specific historical threads are picked up in [11], [16, ch5], and [22] on the history of the divergence theorem and Green's identities; [16, ch4] on mathematics at Cambridge during Green's time; [15] on the early history of solitons; [21] on Green's place in the pre-history of asymptotic methods for solving differential equations; and [13] and [28] enable one to relive the birthpangs surrounding the development of modern vector analysis.

Green's Mill and Centre is situated one mile East of Nottingham city centre and is open from 10 am until 5 pm every day except Mondays and Tuesdays. It is close to other places of significance in Green's life, as shown on the map in figure 8. Various items of Green memorabilia, including copies of the *Essay*, are available at the Mill.

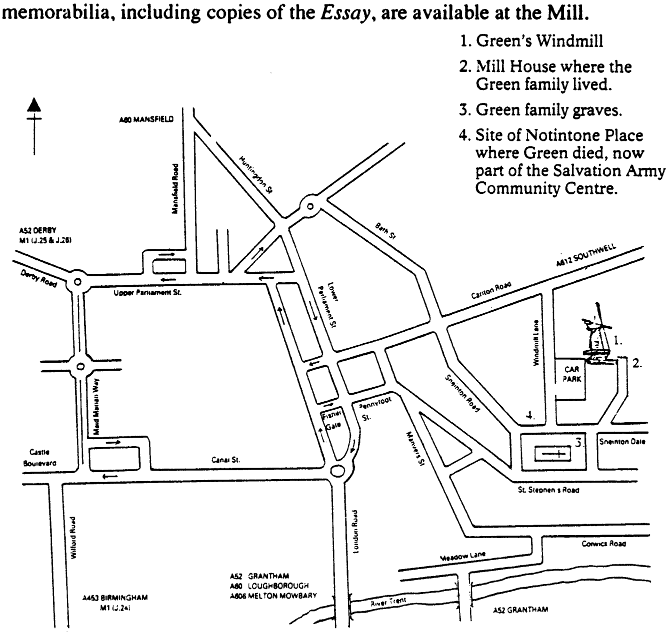


FIGURE 8. Green's Mill and Centre. Belvoir Hill, Sneinton, Nottingham NG2 4LF.

1. Ferrers, N.M. (ed.), *Mathematical papers of George Green*, reprinted by Chelsea (1970).
2. University of Nottingham Libraries, *George Green, miller and mathematician 1793–1841*, (1988). A catalogue of sources, obtainable for £1 from Nottingham University Library.

3. Cannell, D.M., *George Green, mathematician and physicist, 1793–1841: the background to his life and work*, Athlone Press (1993).
4. Cannell, D.M., *George Green, miller and mathematician 1793–1841*, Castle Museum, Nottingham (1988). Obtainable from Green's Mill, £2.50.
5. Challis, L.J., "George Green – miller, mathematician and physicist", *Math. Spectrum* 20 No. 2 (1987/8).
6. Farina, J.E.G., "The work and significance of George Green, the miller mathematician, 1793-1841" *Bulletin of the I.M.A.* 12 (1976) 98–105 .
7. Gray, G.J.. "George Green", *Dictionary of national biography, vol. XXIII*, Smith, Elder & co. (1890).
8. Green, H.G., "A biography of George Green", in Ashley, M.F., (ed.) *Studies and essays in the history of science and learning*, New York (1947).
9. Phillips, D., (ed.) *George Green, Miller, Sneinton*, Castle Museum, Nottingham (1976, out of print).
10. Whitrow, G.J., "George Green (1793–1841): a pioneer of modern mathematical physics and its methodology", *Annali dell'Istituto e Museo di Storia della Scienza di Firenze* IX (1984) 45–68.
11. Archibald, T., "Connectivity and smoke-rings: Green's second identity in its first fifty years", *Mathematics Magazine* 62 (1989) 219–232.
12. Barton, G., *Elements of Green's functions and propagation: potentials, diffusion and waves*, Oxford U.P. (1991).
13. Crowe, M.J., *A history of vector analysis*, Univ. of Notre Dame Press (1967).
14. Dieudonné, J., *History of functional analysis*, North-Holland (1981).
15. Drazin, P.G., *Solitons*, Cambridge U.P. (1983).
16. Harman, P.M. (ed.), *Wranglers and physicists: studies on Cambridge mathematical physics in the nineteenth century*, Manchester U.P. (1985).
17. Kellogg, O.D., *Foundations of potential theory*, Dover (1954).
18. Love, A.E.H., *A treatise on the mathematical theory of elasticity (2nd ed)*, Cambridge U.P., (1906).
19. Ramsey, A.S., *An introduction to the theory of Newtonian attraction*, Cambridge U.P. (1982).
21. Schlissel, A., "The development of asymptotic solutions of linear ODEs, 1817–1920", *Arch. for Hist. of Exact Sciences* 16 (1977) 307–78.
22. Stolze, C.H., "A history of the divergence theorem", *Historia Mathematica* 5 (1978) 437–42.
23. Strauss, W.A., *Partial differential equations – an introduction*, Wiley (1992).
24. Temple, G., *100 years of mathematics*, Duckworth (1981).
25. Thomson, W. and Tait, P.G., *Treatise on natural philosophy, Part I, Part II*, Cambridge U.P. (1879, 1903).
26. Thomson, W., *Reprint of papers on electrostatics and magnetism*, Macmillan (1872).
27. Todhunter, I. and Pearson, K., *A history of the elasticity and strength of materials, vol. I*, Cambridge U.P. (1886).
28. Weatherburn, C.E., *Advanced vector analysis*, Bell (1924).
29. Whittaker, E.T., *A history of the theories of aether and electricity*, reprinted by Dover (1989).

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