Let \( \Gamma = \{(x, y) : 0 < x \leq 1, \ y = \sin(\frac{1}{x})\} \cup \{(0, y) : |y| \leq 1\} \)

**Theorem 1.** \( \Gamma \) is not path connected.

*Proof.* Suppose \( f(t) = (a(t), b(t)) \) is a continuous curve defined on \([0, 1]\) with \( f(t) \in \Gamma \) for all \( t \) and \( f(0) = (0, 0), f(1) = \left(\frac{1}{\pi}, 0\right) \). Then by the intermediate value theorem there is a \( 0 < t_1 < 1 \) so that \( a(t_1) = \frac{2}{3}\pi \). Then there is \( 0 < t_2 < t_1 \) so that \( a(t_2) = \frac{2}{5}\pi \). Continuing, we get a decreasing sequence \( t_n \) so that \( a(t_n) = \frac{2}{2n+1}\pi \). It follows that \( b(t_n) = (-1)^n \). Now since \( t_n \) is a decreasing sequence bounded from below it tends to limit \( t_n \to c \). Since \( f \) is continuous \( \lim_{n \to \infty} f(t_n) \) must exist. But \( \lim_{n \to \infty} b(t_n) \) does not exist. \( \Box \)