Math 334 Sample Problems

One side of one notebook sized page of notes will be allowed on the test. You may work together on the sample problems – I encourage you to do that. The test will cover up to §4.4.

1. Let f be defined and bounded on $[a, b]$. Define a function g on $[a, b]$ by the formula $g(x) = \overline{I}(\chi_{[a,x]} f)$. In other words $g(x)$ is the upper integral of f on the interval $[a, x]$. Prove that g is continuous on $[a, b]$. Suppose f is continuous at x_0. Prove that $g'(x_0) = f(x_0)$. The same is true for lower integrals.

2. Folland, §4.1, # 9.

3. Suppose $a < b < c < d$. Let $I = [a, b],\ J = [c, d],\ R = I \times J$ and let $f(x, y) = |x - y|$, if $x \in I,\ y \in J$. Compute $\displaystyle\int_R f$.

4. Using the method of Lagrange multipliers, find the highest and lowest points of the circle
$$x^2 + y^2 + z^2 = 16,\ (x + 1)^2 + (y + 1)^2 + (z + 1)^2 = 27$$

5. Show that the surface $z = 3x^2 - 2xy + 2y^2$ lies entirely above every one of its tangent planes. Hint: Look at the Taylor expansion at every point.

6. Let $f(x,y) = \sec(x + y^2)$. Find the first two non-zero terms in the Taylor series of $\cos x$, centered at 0. Use it to find the first two non-zero terms of the Taylor series of $\sec x$ centered at 0. Then use that series to find the first two non-zero terms of f at $(0,0)$.

7. Let g be a polynomial of degree three. Prove that

$$\int_{-1}^{1} g = \frac{g(-1) + 4g(0) + g(1)}{3}.$$

8. Consider the following function

$$F(x,y) = (\frac{x}{1+x+y}, \frac{y}{1+x+y}),$$

which has the set $\{(x,y) : 1 + x + y \neq 0\}$ as its domain. Compute $\dfrac{\partial(f,g)}{\partial(x,y)}$. Where is it different from 0? Show that F is $1-1$ and find an explicit formula for its inverse. Use these results to describe the exact image of F

9. Folland, §2.9, problem 16.

10. Let f be a positive continuous function on $I = [a,b]$. Let $M = \max\{f(x) : x \in I\}$. Prove that

$$\lim_{n\to\infty} \left(\int_I f^n \right)^{1/n} = M.$$

11. Suppose $F(x,y)$ is a C^2 function that satisfies the equations $F(x,y) = F(y,x), F(x,x) = x$. Prove that the quadratic term in the Taylor polynomial of F based at the point (a,a) is $\frac{1}{2}F_{xx}(a,a)(x-y)^2$.

12. There may be homework problems or example problems from the text or lectures on the midterm.