Many of you are unfamiliar with hyperbolic functions. Here is a crash course on hyperbolic functions. Trigonometric functions could be called circular functions since \(( \cos t, \sin t )\) is a parameterization of the circle \(x^2 + y^2 = 1\). Similarly \(( \cosh t, \sinh t )\) is a parameterization of the hyperbola \(x^2 - y^2 = 1\) and hence \(\sinh t, \cosh t\) are referred to as hyperbolic functions. The functions \(\sinh t, \cosh t\) are defined as follows.

\[
\cosh t = \frac{e^t + e^{-t}}{2} \\
\sinh t = \frac{e^t - e^{-t}}{2}
\]

It follows that

\[
\cosh^2 t - \sinh^2 t = 1.
\] (1)

It is also easy to see that

\[
\cosh(s + t) = \cosh(s) \cosh(t) + \sinh(s) \sinh(t), \quad (2)
\cosh(2t) = \cosh^2(t) + \sinh^2(t) \quad (3)
\]

\[
= 2 \cosh^2(t) - 1, \quad (4)
\sinh(s + t) = \sinh(s) \cosh(t) + \sinh(t) \cosh(s), \quad (5)
\sinh(2t) = 2 \sinh(t) \cosh(t). \quad (6)
\]

Also

\[
\frac{d}{dt} \cosh t = \sinh t, \quad (7)
\frac{d}{dt} \sinh t = \cosh t. \quad (8)
\]

These functions can come in handy in integration problems. For example let us find an antiderivative of \(\sqrt{1 + x^2}\). We substitute \(x = \sinh t\) to get

\[
\int \sqrt{1 + x^2} \, dx = \int \cosh^2(t) \, dt \\
= \int \frac{\cosh(2t) + 1}{2} \, dt \\
= \frac{t}{2} + \frac{\sinh(2t)}{4} \\
= \frac{t + \sinh(t) \cosh(t)}{2} \\
= \frac{x\sqrt{1 + x^2} + \sinh^{-1}(x)}{2}.
\]
By using the quadratic formula we see that
\[ \sinh^{-1}(x) = \log(x + \sqrt{1 + x^2}), \]
and hence
\[ \int \sqrt{1 + x^2} \, dx = \frac{x\sqrt{1 + x^2} + \log(x + \sqrt{1 + x^2})}{2}. \]