1. Eilenberg-Maclane spaces, continued. We still need to show that Eilenberg—Maclane spaces represent cohomology.

2. Cofibrations. There is hardly anything new here, as a cofibration $A \rightarrow X$ is essentially the same thing as inclusion of a subspace such that $(X, A)$ has the homotopy extension property; see Hatcher Proposition 4H.1. On the other hand, the shift in point of view from subobjects to morphisms will prove very useful. Basic facts and concepts to be considered include:

- Cofibrations are closed under composition and pushout.
- Every map $f : X \rightarrow Y$ can be factored as $X \rightarrow Z \rightarrow Y$ where the first map is a cofibration and the second a homotopy equivalence. (This isn’t new; we just take $Z = M_f$ to be the mapping cylinder of $f$, together with the usual maps.)
- Homotopy cofibers and cofiber sequences; the Barratt-Puppe sequence (cofiber version; see Hatcher p. 398).

3. Fibrations. References include Hatcher and my notes on fibrations posted online.

A map $f : X \rightarrow Y$ is a fibration, or Hurewicz fibration, if it has the homotopy lifting property (Hatcher p. 375) for all spaces $W$. For many purposes it is convenient to use the weaker notion of Serre fibration, in which the lifting property is assumed only for CW-complexes $W$. One can think of the concept “fibration” as extracting the homotopy-theoretic essence of the concept “local product” or “fiber bundle”. In particular every local product is a fibration (this is much easier to prove for Serre fibrations). Basic facts and concepts to be considered include:

- Fibrations are closed under composition and pullback.
- Every map $f : X \rightarrow Y$ can be factored as $X \rightarrow Z \rightarrow Y$ where the first map is a homotopy equivalence and the second a fibration.
- Homotopy fibers and fiber sequences; the Barratt-Puppe sequence (fiber version; Hatcher p. 409).

- Loop spaces and the path-loop fibration.

- The long exact sequence on homotopy groups associated to a fibration (“dual” to the long exact sequence on homology/cohomology groups associated to a cofibration).

**Remark:** Quillen introduced an axiomatic approach to homotopy theory, in which one starts from a “closed model category”. This is a category equipped with three distinguished classes of morphisms called fibrations, cofibrations and weak equivalences, subject to suitable axioms. Although we will not use model categories, many of the basic properties of fibrations and cofibrations that we consider either motivate, or in some cases are motivated by, the model category approach. The property I call the “main lifting property” in my fibration notes is in this spirit.

**Remark:** Since we will be considering path spaces, loop spaces, and other spaces of continuous maps, we’ll need the basic facts about the compact-open topology on function spaces. As this would be a very dull topic for the lectures, I’ll just assume results as needed (see the Appendix to Hatcher for details).

4. Principal bundles and fiber bundles with structure group. See my notes on principal bundles, which will be the main reference.

5. Characteristic classes. References include the Milnor-Stasheff text and lecture notes. We will compute the cohomology of the classifying spaces $BU(n)$, $BO(n)$, define Chern, Stiefel-Whitney and Pontrjagin classes, establish the connection with symmetric functions, and more. All this will be straightforward with the machinery we’ll have in place.

6. **Further topics.** Toward the end I’d like to give a few “big picture” expository lectures on various topics, as well as have some more lectures by you (the ones you’ve given so far have been very interesting!). Here are some of the topics I have in mind; as usual feel free to come up with your own. Many of these are interlinked and will need to be coordinated and subcontracted in some way.
   a. Cobordism.
   b. K-theory.
   c. Stable homotopy theory.
   d. Spectral sequences.

Regarding spectral sequences, there would be little point in going through the technical details of the constructions in class; it would be too tedious and time-consuming. I have in mind presenting my own take on how to think about and use the Serre spectral sequence, in a way that applies to many other spectral sequences as well. In any case, the Serre spectral sequence alone will augment our computational power (i.e. for computing homology and cohomology rings) by several orders of magnitude.