HW #2, DUE 4-14

MATH 506A

1. Exercise 7.19, p. 209, of Eisenbud.

2. Exercise 7.20, p. 209, of Eisenbud.

3. For p a prime number, show that p-adic completion is *not* right exact for general $\mathbb{Z}$-modules. (Use the example given in class; right exactness fails in the *middle* of the sequence, not its right end.)

4. Let C be the ring of continuous real-valued functions on the real line $\mathbb{R}$. Show that the 0 ideal of this ring has no primary decomposition, i.e. it is not the intersection of finitely many primary ideals. You may use Urysohn's Lemma in this context, which asserts for any disjoint closed subsets F_1, F_2 of $\mathbb{R}$, there is a function in C which equals 0 on F_1 and 1 on F_2.

5. Let $P_1, \dots, P_n$ be finitely many ideals in a ring R such that at most two of the P_i are not prime. Show that any ideal I contained in the union of the P_i is contained in just one of them. Also give an example of an ideal I of $k[x.y]$, k a field, contained in a countable union of prime ideals without being contained in any one of them.