SAMPLE FINAL PROBLEMS–MATH 506A

1. State a version of Hilbert's Nullstellensatz that holds over an arbitrary basefield k.

2. Give a precise statement of the rational canonical form for square matrices over a field k.

3. Classify the prime ideals in the ring $\mathbb{Z}[i]$ of Gaussian integers.

4. Give an example of a Noetherian domain of dimension one that is not a PID.

5. Compute the cohomology groups $H^n(\mathbb{Z}_m, A)$ of the cyclic group $G = \mathbb{Z}_m$ of order m with coefficients in any G-module A.

6. Prove or disprove: two finite groups G, H are isomorphic if and only if their complex group algebras $\mathbb{C}G, \mathbb{C}H$ are isomorphic as rings.

7. The ring $R = \mathbb{Z}[\sqrt{-7}]$ is not quite a Dedekind domain; how can this ring be enlarged to make it such a domain?

8. Compute the Krull dimension of the polynomial ring $\mathbb{Z}[x, y, z]$.