Sample Final Problems–Math 506

1. State the version of the Nullstellensatz that holds for arbitrary basefields.

2. Let $f : A \to B$ be a homomorphism of rings. Show that the induced map $f^* : \mathrm{Spec}(B) \to \mathrm{Spec}(A)$ is continuous with respect to the Zariski topology on both spectra.

3. State the theorem on existence of primary decompositions of ideals in a Noetherian ring, indicating also the two features of this decomposition that are uniquely determined by the ideal. Also give the definition of a primary ideal.

4. Find all ideals in the power series ring $k[[x]]$, where k is a field..

5. Give a sufficient condition for a polynomial $q \in \mathbb{Z}[x]$ to have a solution in $\mathbb{Z}_p$, the ring of p-adic integers, for some prime p.

6. Give an example of a non-projective flat module over a commutative ring..

7. Give the definition of a the dimension of a Noetherian local ring in terms of the Hilbert (or characteristic) polynomial attached to its maximal ideal.

8. Compute the dimension of the tangent space at all points of the curve in $\mathbb{C}^2$ defined by the equation $y^2 = x^3 + x^2$.