FINAL EXAM

1. Find two examples of a ring R with exactly two prime ideals, one with one such ideal contained in the other, the other with neither ideal contained in the other.

2. Classify as completely as you can the fields that are homomorphic images of $\mathbb{Q}[x, y, z]$, the polynomial ring in three variables over $\mathbb{Q}$.

3. Find all singular points on the affine curve in $\mathbb{C}^2$ defined by the equation $y^2 = x^3 + x^2$ and find the dimension of the tangent space to this curve at each such point.

4. Let k be a field. Find a primary decomposition of the ideal $I = (xy, y^2)$ in the polynomial ring $k[x, y]$ in two generators over k; that is, realize I as a finite intersection of primary ideals. Also identify the radicals of each of these ideals.

5. If K is an algebraically closed field, classify the subvarieties of K^n whose coordinate rings are Artinian.

6. The example of the curve C in $\mathbb{C}^2$ defined by the equation $x^3 = y^2$ has been discussed multiple times in class. State as many properties of this curve and its coordinate ring as you can, focussing on the ones that make it different from most curves and their coordinate rings.

7. Show that the quotient field K of an integral domain R is flat as an R-module.

8. Let k be an algebraically closed field, A an affine domain over k. We have computed the dimension of A in four different ways (three of them involving a choice of maximal ideal M of A, but all three giving the same answer for any given M). Describe these ways as clearly as you can (but without giving proofs).