

Optional Problems

Math 506A

1. Let V be the union of the coordinate axes in \mathbb{A}^2 . Exhibit the coordinate ring $k[V]$ explicitly as an integral extension of a polynomial ring. What is the size of the generic fiber of the associated morphism from V to \mathbb{A}^1 ?
2. Let k be an algebraically closed field and let P be the ideal $(y^2 - x^3 - x^2)$ generated by $y^2 - x^3 - x^2$ in $k[x, y]$. Show that P is prime and compute the integral closure of $k[V]$, the coordinate ring of the corresponding variety V , in its quotient field.
3. DF, Exercise 6, p. 703.
4. Let S be an integral extension of a ring R , D a multiplicatively closed subset of R , Show that $D^{-1}S$ is integral over $D^{-1}R$.