HW #8

Math 506A

1. Let V be an algebraic subset of $\mathbb{C}^n$. Use a consequence of Noether normalization proved in class to show that V is compact in the *Euclidean* topology if and only if it is finite.

2. (20 points) Let I be the principal ideal (2) in the ring $R = \mathbb{Z}[\sqrt{-3}]$. Show that there is only one proper prime ideal P of R containing I and identify P. Show that I lies strictly between P and P^2 and deduce that I is not a product of prime ideals in R.

3. DF, Exercise 6, p. 703.

4. Let $V = Z(xz-y^2, yz-x^3, z^2-x^2y)$, the zero locus in $\mathbb{A}^3$ of the given polynomials. Show that the morphism from $\mathbb{A}^1$ to V sending t to (t^3, t^4, t^5) is bijective, by setting $t = y/x$ on V if $x \neq 0$. Show nevertheless that V is not isomorphic to $\mathbb{A}^1$, by showing that V has a singular point.