HW #1

Math 506A

1. Show that the characteristic polynomial of the companion matrix $C(f)$ is f (up to sign) for any monic polynomial f.

2. Show that the number of similarity classes of 3×3 matrices over $\mathbb{Q}$ with a given characteristic polynomial is the same as the number of similarity classes of such matrices over any extension field of $\mathbb{Q}$. Give an example to show that this is not true in general for 4×4 matrices.

3. Show that a square matrix over k is diagonalizable if and only if its minimal polynomial over k is the product of distinct linear factors.

4. (20 points) Let $q = p^n$ be a power of a prime p. A *deBruijn sequence for q* is a sequence $s = a_0 \dots a_{q-1}$ of q symbols, each an element of the field F_p of order p, such that every sequence $b_1 \dots b_n$ of n symbols, each an element of F_p, occurs exactly once as a consecutive subsequence $a_i \dots a_{i+n-1}$ of s for $i \in [0, q-1]$; here addition of subscripts takes place modulo q, so that the sequence wraps around. Show that a deBruijn sequence exists by the following steps. Start with the field F_q of order q and show that it has a nonzero element x of multiplicative order $q-1$. Let f be the minimal polynomial of x over F_p, so that f has degree n. Then the matrix of multiplication by x on F_q, regarded as an n-dimensional vector space over F_p, is the transpose $M = C(f)^t$ of $C(f)$ with respect to some basis, by a result in class. Now define the sequence s by decreeing first that $v = (a_0, \dots, a_{n-1}) = (0, \dots, 0, 1)$ and then that the vector $(a_i, \dots, a_{i+n-1}) = (M^i v^t)^t$, the transpose of the column vector $M^i v^t$, for $0 \le i \le q-n$. Show that this makes sense and yields a deBruijn sequence $a_0 \dots a_{q-1}$, making sure to check also that $00 \dots 0$ is a consecutive subsequence of s.