HW #7, due 5-16

Math 506A

1. (20 points) Exercise 7.4.33, DF, p. 259 (hint: use compactness in part (a))

2. (20 points) Exercise 15.2.45, DF, p. 690.

3. (10 points) Let p be a prime number and let A be a countable direct sum of copies of $\mathbb{Z}/p\mathbb{Z}$. Let B the direct sum $\oplus_{n=1}^{\infty}\mathbb{Z}/p^n\mathbb{Z}$. For each n let α_n be the map from the nth copy of $\mathbb{Z}/p\mathbb{Z}$ in A to $\mathbb{Z}/p^n\mathbb{Z} \subset B$ sending 1 to p^{n-1}; let α be the sum of the α_n. Show that the p-adic completion of A is A itself while the p-adic completion of A for the topology induced by the p-adic topology on B is the direct *product* of countably many copies of $\mathbb{Z}/p\mathbb{Z}$.