HW #5 due 5-2

Math 506A

1. Give an example of a non-Noetherian ring R such that Spec R is finite.

2. Let A, B be Noetherian rings with $A \subset B$. We say that B has the going-up property with respect to A if given prime ideals P_1 of A and Q_1 of B with $Q_1 \cap A = P_1$ and a second prime ideal P_2 of A containing P_1, there is a prime ideal Q_2 of B with $Q_2 \cap A = P_2$. Show that B has this property if and only if the map Spec $B \to$ Spec A is closed (takes closed sets to closed sets), using the finiteness of the number of minimal primes over any ideal of A or B.

3. Let P^r, P^s respectively denote projective r- and s-space. Find an embedding of the product $P^r \times P^s \to P^N$ for suitable N and show that the image of this map is a subvariety of P^N.

4. Dummit and Foote, Exercise 15.5.8, p. 746.

5. Exercise 15.5.12.

Skim the first chapter of Hartshorne.