HW #8, DUE 3-3

1. Let A be a commutative ring. Show that $p \in A[x], p \neq 0$ is a zero divisor in $A[x]$ if and only if that is $a \in A, a \neq 0$ with $ap = 0$. (Look at a polynomial q of least possible degree such that $pq = 0$). Generalize this result to $A[x_1, \dots, x_n]$.

2. Let K be an algebraically closed field of characteristic 0 and let $V \subset K^2$ be our old friend the variety with equation $x^2 = y^3$. Show that the maximal ideal $M_{(1,1)}$ of the coordinate ring $K[V]$ at the point $(1, 1)$ is not the radical of any principal ideal.

3. Again let K be algebraically closed (but of arbitrary characteristic). Show that the subvariety of K^2 with equation $y^3 = x^5$ has a singularity at the origin that is resolved by two blowups.

4. Let C be the rational quartic curve in $\mathbb{P}^3$, consisting of all points with projective coordinates (s^4, s^3t, st^3, t^4) for some $s, t \in K$, s, t not both 0. Find a set of four generators of the (homogeneous) ideal I of C and prove that I cannot be generated by three elements.

5. Let A be a commutative ring, I an ideal of A. Give A the standard I-filtration, taking $A_n = I^n$, and assume that $\cap_n I^n = 0$ and that the graded ring $G = G(A)$ is an integral domain. Show that A is also an integral domain.