Sample Final Problems–Math 505

1. Let $E \supset F$ be a Galois extension of degree 6. What are the possibilities for the Galois group G of E over F? For each such possibility, determine the number of intermediate fields between E and F and work out the correspondence between intermediate fields and subgroups of G explicitly.

2. Sketch the classification of finite fields. When is a finite field a Galois extension of one of its subfields?

3. How can one modify the ring $\mathbb{Z}[\sqrt{-3}]$ slightly to make it a Dedekind domain?

4. Determine all quotients of $\mathbb{C}[x, y, z]$ that are fields.

5. Show that affine space K^n and projective space $\mathbb{P}^n$ are not homeomorphic for $n > 2$ (repeat of HW problem).

6. Define the integral closure of an integral domain A (in its quotient field K).

7. Classify the prime ideals in the power series ring $K[[x]]$ for any field K.

8. Define the strict transform (or blowup) of an affine variety.