SAMPLE FINAL PROBLEMS–MATH 505A

1. Work out the Galois correspondence for the splitting field of $x^3 - 2$, regarded as a Galois extension of $\mathbb{Q}$, explicitly identifying all the fields and groups arising in this correspondence. Also work out the correspondence for the *non-Galois* extension $\mathbb{Q}[2^{1/3}]$ of $\mathbb{Q}$.

2. The subring $R = \mathbb{Z}[\sqrt{-11}]$ of $\mathbb{C}$ generated by $\mathbb{Z}$ and $\sqrt{-11}$ is not quite a Dedekind domain. How can one slightly modify R so as to make it a Dedekind domain?.

3. Give a careful definition of the cohomology groups $H^n(G, A)$ attached to a finite group G and a $\mathbb{Z}$-module A on which G acts linearly.

4. State Hilbert's Theorem 90 for finite Galois extensions of fields with cyclic Galois groups.

5. Give an example of a commutative Artinian ring with exactly seven maximal ideals.

6. Give a simple sufficient condition for an irreducible polynomial of degree 5 over $\mathbb{Q}$ to have Galois group S_5 (and so not to be solvable by radicals).

7. Show that -1 is a square in the ring Z_5 of 5-adic integers.

8. Let D be a central division algebra over $\mathbb{Q}$ of degree 25. Find the degree of a maximal subfield K of D over $\mathbb{Q}$.