HOMEWORK #7, DUE 11-18

MATH504A

1. Show that a finite group G is abelian if and only if its irreducible representations all have degree 1.

2. Construct the character tables of the group G of quaternion units and the dihedral group H of order 8 and verify that they are the same.

3. The purpose of the remaining three problems is to show that any two nonabelian groups of order p^3, p a prime, have identical character tables. Let G be a nonabelian group of order p^3. Show that its center Z is cyclic of order p (say generated by z) and that G/Z is isomorphic to the direct product $\mathbb{Z}_p \times \mathbb{Z}_p$. Also show that G has p conjugacy classes of size 1 and $p^2 - 1$ such classes of size p, so $p^2 + p - 1$ conjugacy classes altogether.

4. Deduce that G has p^2 distinct representations of degree 1, on each of which Z acts trivially. Show that the conjugacy class of any $g \in G$ with $g \notin Z$ takes the form $\{g, gz, \ldots, gz^{p-1}\}$.

5. In each of the remaining irreducible representations of G, the generator z of Z acts by a complex p-th root of 1 different from 1 itself. Deduce that there are $p - 1$ such representations, each of degree p. Show that the character of each of them on any $g \notin Z$ equals 0. Work out the character table of G from this and observe that it does not depend on the structure of G (apart from G being nonabelian).