HOMEWORK #4, DUE 10/28

MATH 504A

1. Show that any square matrix M over any field is similar to its transpose M^t.

2. Show that, if the square matrices M, N over the field K are similar over some larger field L (that is, conjugate via an invertible matrix over L), then they are already similar over K.

3. Classify the finitely generated projective modules over a PID R.

4. Let M be a free module of rank n over a commutative ring R. Show that the kth exterior power $\bigwedge^k M$ is also free over R and determine its rank, by exhibiting an explicit basis for it in terms of a basis $b_1, \dots, b_n$ of M. Use determinants to show that your basis is linearly independent over R.

5. Show that any $\mathbb{Z}$-module M injects (there is an injective from M) into an injective $\mathbb{Z}$-module N.