HW #2, due 10-11

Math 504A

1. Let G be the free group on two generators x, y. Find a free basis for a normal subgroup N such that G/N is cyclic of order two.

2. Let K be a field. Denote by K^∞ the vector space of sequences $(k_1, k_2, \dots)$ with $k_i \in K$ and all but finitely many k_i equal to 0. Let R be the ring End K^∞ of endomorphisms of R. Show that $R \cong R \oplus R$ as an R-module, so that the rank of R as a free module over itself is not well defined.

3. Let R be a commutative ring and M an $n \times n$ matrix over R, so that M defines an endomorphism T of the free R-module R^n. Show that T is surjective if and only if it is an isomorphism, or if and only $\det M$ is a unit in R. Deduce that there is no surjective R-module homomorphism from R^m to R^n if $m < n$.

4. In the setting of the last problem, show that T is injective if and only if $\det M$ is not a zero divisor in R.

5. Show that the free groups $F_s . F_t$ of ranks s, t, respectively, are isomorphic if and only if $s = t$, by showing that the quotients of F_s, F_t by their respective commutator subgroups are not isomorphic if $s \neq t$.

Also skim section 6.3 and read sections 10.1-3 in the text.