SAMPLE FINAL PROBLEMS–MATH 504A

1. Give a careful definition of an injective module M over a ring R..

2. Give a precise statement of the rational canonical form for square matrices over a field k.

3. Prove or disprove: two finite groups G, H are isomorphic if and only if their group algebras $\mathbb{C}G, \mathbb{C}H$ are isomorphic as rings.

4. It was shown in class that the character table of any symmetric group S_n consists entirely of integers. Does this result extend to the alternating groups A_n?

5. Show that a module over the polynomial ring $R = K[x]$ is generated by two elements if and only if it is the direct sum of two quotients of R. Here K is a field and R itself is regarded as a quotient of R.

6. Classify the finitely generated projective modules over $\mathbb{Z}$.

7. Give an example of two nonisomorphic finite groups with isomorphic character tables.

8. Suppose that a finite group G has a subgroup H of index m such that the order $|G|$ of G does not divide $m!$ Show that G has a nontrivial normal subgroup.