Study problems for final exam, not to turn in

Math 327C/574C

1. Give an example of a series which converges conditionally by the Dirichlet Test, but to which the Alternating Series Test does not apply.

2. Give the definition of the product of two cuts C_1, C_2 defining positive numbers.

3. Write down a power series for $\int_0^x \frac{1}{1+t^4}\,dt$ and determine its radius of convergence.

4. Does the geometric series $\sum_{n=0}^{\infty} x^n$ converge uniformly to its sum on the open interval $(0,1)$? Why or why not?

5. Verify the addition law for the sine function ($\sin(x+y) = \sin x \cos y + \cos x \sin y$) using the power series definitions of $\sin x$ and $\cos x$.

6. Suppose that the graph of the function $y = f(x)$ crosses both the line $y = x$ and the x-axis infinitely many times on any closed interval $[0, \alpha]$ for $\alpha > 0$. Show that $f'(0)$ does not exist.

7. Let $f(x)$ be a continuous function from the interval $[0,1]$ to itself. Show that that $f(x) = x$ for some $x \in [0,1]$.