## Study problems for second midterm, not to turn in

## Math 327C/574C

1. Use the Dirichlet Test to prove the Alternating Series Test for convergence of a series.

2. Prove or disprove: if  $f : \mathbb{R} \to \mathbb{R}$  is continuous and  $C \subset \mathbb{R}$  is compact, then the inverse image  $f^{-1}(C)$  is also compact. (That is, if this statement is true, then prove it; if not, then give a counterexample.)

3. Determine the radius of convergence of the power series  $\sum_{n=1}^{\infty} \frac{x^n}{n^3}$  and show that it defines a continuous function on a suitable interval in the real line. Do not try to evaluate the sum.

4. Define carefully what it means for a sequence  $f_n$  of real-valued functions on interval [a, b] to converge uniformly to a function f on [a, b]. If the  $f_n$  are continuous, then what can you say about f?

5. Multiply the power series for  $e^x$  by the series for  $e^y$  and verify that you get the series for  $e^{x+y}$ .

The midterm will cover the convergence test for series and Chapters 3-6.