MIDTERM #2

Math 327C/574C

name

Show all work; doing this may help you get more partial credit for problems done incorrectly. Use the backs of the test pages as necessary.

1. State the basic properties proved in class about continuous functions on a closed bounded interval and indicate which of these also hold for derivatives on such an interval.

2. Let a_k be a periodic sequence of signs, so that there is an integer $N > 0$ with $a_{k+2N} = a_k$ for all k and each $a_k = \pm 1$. Assume also that $\sum_{k=1}^{2N} a_k = 0$ and let b_k be a sequence of real numbers such that $b_k \geq b_{k+1}$ for all k and $b_k \to 0$ as $k \to \infty$. Use a theorem in class to show that the *series* $\sum_{k=1}^{\infty} a_k b_k$ converges.

3. Work out a power series expansion of $g(x) = e^{x^3}$, by starting with a power series for e^x and then making a suitable change of variable.

4. Correct the following *misstatements* of theorems proved in class (you need not prove the corrected versions).

(a) If the pointwise limit f of a sequence of continuous functions f_n is continuous, then the convergence is uniform.

(b) If f is a continuous function on $\mathbb{R}$ and $C \subset \mathbb{R}$ is connected, then the inverse image $f^{-1}(C)$ is connected.

(c) If $f : \mathbb{R} \to \mathbb{R}$ is infinitely differentiable (i.e. has derivatives of all orders), then f has a Taylor expansion $f(x) = \sum_{n=0}^{\infty} a_n x^n$ with a positive radius of convergence.

5. Give an example (possibly using a theorem in class) of a function f that is the uniform limit of differentiable functions on an interval $[a, b]$ but is not differentiable on that interval.