MIDTERM #2 SOLUTIONS

1. For each of the following series, determine whether or not it converges, briefly justifying your answer in each case.

(a) $\sum_{n=1}^{\infty} \frac{n^n}{n!}$

Diverges by nth term test (terms do not go to 0). (b) $\sum_{n=1}^{\infty} \frac{\cos n}{n^3}$

Converges absolutely by comparison with p-series, $p = 3$ (c) $\sum_{n=1}^{\infty} \frac{n^3+4n+6}{n^4+5n+10}$

Diverges by limit comparison with harmonic series.

2. State the three basic theorems proved in class about continuous functions on a closed bounded interval and indicate which of these also hold for derivatives on such an interval.

Boundedness Theorem: any such function is bounded. Extreme Value Theorem: any such function has both a maximum and a minimum. Intermediate Value Theorem: any such function takes on all values between its maximum and minimum. Only the second of these holds for derivatives.

3. Work out a power series expansion of $g(x) = e^{x^2}$, by starting with a power series for e^x and then making a suitable change of variable.

Replacing x by x^2 in the series for e^x, we get $\sum_{n=0}^{\infty} \frac{x^{2n}}{n!}$

4. Correct the following *misstatements* of theorems proved in class (you need not prove the corrected versions).

(a) If the pointwise limit f of a sequence of continuous functions f_n is continuous, then the convergence is uniform.

If f_n converges uniformly and the f_n are continuous, then so is their limit.

(b) If a sequence a_n of nonnegative real numbers converges to 0, then the alternating series $\sum_{n=0}^{\infty}(-1)^n a_n$ converges.

If a_n is a sequence of nonnegative rela numbers with $a_n \geq a_{n+1}$ for all n and $a_n \to 0$ as $n \to \infty$, then $\sum_{n=0}^{\infty}(-1)^n a_n$ converges. (c) A differentiable function f defined on a subset S of $\mathbb{R}$ is constant on S if and only if its derivative is 0 on S.

A differentiable function f defined on an interval I is constant on I if and only if its derivative is 0 on I.

5. Show that the series $\sum_{n=1}^{\infty} \frac{x^n}{n^3}$ defines a continuous function $f(x)$ for $x \in [-1, 1]$. Do NOT try to find a formula for $f(x)$.

The series converges uniformly by the Weierstrass M-test; its sum is thus continuous since its terms are.