MIDTERM #2

Math 327

name

Show all work; doing this may help you get more partial credit for problems done incorrectly. Use the backs of the test pages as necessary.

1. For each of the following series, determine whether or not it converges, briefly justifying your answer in each case.

(a) $\sum_{n=1}^{\infty} \frac{n^n}{n!}$

(b) $\sum_{n=1}^{\infty} \frac{\cos n}{n^3}$

(c) $\sum_{n=1}^{\infty} \frac{n^3+4n+6}{n^4+5n+10}$

2. State the three basic theorems proved in class about continuous functions on a closed bounded interval and indicate which of these also hold for derivatives on such an interval.

3. Work out a power series expansion of $g(x) = e^{x^2}$, by starting with a power series for e^x and then making a suitable change of variable.

4. Correct the following *misstatements* of theorems proved in class (you need not prove the corrected versions).

(a) If the pointwise limit f of a sequence of continuous functions f_n is continuous, then the convergence is uniform.

(b) If a sequence a_n of nonnegative real numbers converges to 0, then the alternating series $\sum_{n=0}^{\infty}(-1)^n a_n$ converges.

(c) A differentiable function f defined on a subset S of $\mathbb{R}$ is constant on S if and only if its derivative is 0 on S.

5. Show that the series $\sum_{n=1}^{\infty} \frac{x^n}{n^3}$ defines a continuous function $f(x)$ for $x \in [-1,1]$. Do NOT try to find a formula for $f(x)$.