MIDTERM #1

Math 327

name

Show all work; doing this may help you get more partial credit for problems done incorrectly. Use the backs of the test pages as necessary.

1. Show carefully, using the definition of limit, that $\frac{3}{5n^2+3} \to 0$ as $n \to \infty$.

2. The following are *incorrect* versions of theorems proved in class. In each case give the *correct* statement of the theorem.

(a) A bounded sequence converges if and only if it is monotone.

(b) Every nonempty set of real numbers that is bounded below has a least lower bound.

(c) If $f(x)^2$ is continuous at $x = a$, then so is $f(x)$.

3. Let S be a nonempty set of real numbers that is bounded below and $\{C_x : x \in S\}$ be the corresponding set of cuts of rational numbers. Work out an expression for C, the cut corresponding to the greatest lower bound of S, in terms of the C_x.

4. Show that $\sum_{i=1}^{\infty} \frac{\cos^2 i}{2^i}$ converges, by showing that its partial sums are bounded.

5. Let g be a continuous function on $\mathbb{R}$ sending the interval $I = [0, 1]$ to itself. Use the Intermediate Value Property to show that there is $x \in [0, 1]$ with $g(x) = x$.