**FINAL EXAM**

**Math 327B**

name

Show all work; doing this may help you get more partial credit for problems done incorrectly. Use the backs of the test pages as necessary.

1. Show that there is no continuous function $g$ on the interval $[0,1]$ such that $g(\frac{1}{2n}) = \frac{1}{3}$ while $g(\frac{1}{2n+1}) = \frac{1}{2}$ for all positive integers n.

2. Let $f$ be a differentiable function on the unit interval $[0,1]$ such that $f(\frac{1}{n}) = 0$ for all positive integers $n$. Show that there are infinitely many $x \in [0,1]$ such that $f'(x) = 0$.

3. Work out a power series expansion of $\int_0^x e^{-t^3}\,dt$. For which $x$ does this series converge?

4. Let $f(x) = 1/|n|$ if $x = m/n$ is rational, with $m, n \in \mathbb{Z}$ and $m/n$ in lowest terms. Let $f(x) = 0$ is $x$ is irrational. Determine whether or not $f$ is integrable on $[0, 1]$ and compute the integral $\int_0^1 f(x)\, dx$ if it exists.

5. The following statements are either theorems proved in class, or else *misstatements* of such theorems. Correct each, or mark it as already correct.

(a) If the derivative of a function $f$ is nonnegative at a point $c$, then $f$ is weakly increasing in some interval $[a, b]$ containing $c$.

(b) If $\sum |a_n|$ converges, then so does $\sum a_n$.

(c) The series $\sum_{n=1}^{\infty} x^{-n}$ converges for $|x| < 1$.

6. Determine the set of positive real numbers $k$ such that $\sum_{n=0}^{\infty} \frac{1}{1+n^k}$ converges. Indicate which tests you are using to determine this set.

7. Evaluate $\lim_{n\to\infty} \frac{1}{n} \sum_{i=0}^{n-1} e^{\frac{i+\frac{1}{2}}{n}}$, by realizing this sum as a Riemann sum of a suitable function.

8. Show that there is $x \in \mathbb{R}$ with $\cos x = x$.