MIDTERM #2

Math 136

name

Show all work; doing this may help you get more partial credit for problems done incorrectly. Use the backs of the test pages as necessary.

1. Let V and W be finite-dimensional vector spaces and f a linear transformation from V to W. Show that there are bases B, B' of V, W, respectively, such that the matrix of f relative to these bases has at most one 1 in every row and column and all other entries 0. (To construct the basis B, start with a basis for the kernel of f and expand it to a basis of V. Apply f to certain vectors in B to get part of the basis B').

2. Determine the condition on $a \in \mathbb{R}$ for the matrix $A = \begin{pmatrix} 2 & 3 & 1 \\ 3 & 5 & 1 \\ 1 & 1 & a \end{pmatrix}$ to be positive definite.

3. Find the centroid of that portion of the unit ball in $\mathbb{R}^3$ lying in the first octant (i.e. with all coordinates nonnegative). Assume the ball has constant density 1.

4. A plate occupying the elliptical disk defined by $(x^2/a^2) + (y^2/b^2) \leq 1$ is such that its density at any point is proportional to the square of the distance between that point and the origin. Denoting the proportionality constant by k, use a modification of polar coordinates to compute the mass of the plate.

5. Find the equation of the best least-squares linear fit to the data points $(0, 1), (1, 2)$, and $(2, 4)$.