MIDTERM #1

Math 136

name

Show all work; doing this may help you get more partial credit for problems done incorrectly. Use the backs of the test pages as necessary.

1. A manufacturer wants to make cylindrical tin cans with the maximum possible volume for a fixed surface area (measuring the amount of material required to make them). What ratio should the heights of the cans have to their radii to realize this maximum volume? Recall that the volume and surface are of a cylinder of radius r and height h are given by $\pi r^2 h$ and $2\pi rh + 2\pi r^2$, respectively.

2. For every nonzero $a, b, c \in \mathbb{R}$, show that the vectors $(a, a, 0)$, $(b, 0, b)$, and $(0, c, c)$ form a basis of $\mathbb{R}^3$.

3. Find all critical points of the function $f(x, y, z) = xyz + x + y + z$ in $\mathbb{R}^3$.

4. Use row and column operations to bring the symmetric matrix $A = \begin{pmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{pmatrix}$ to diagonal form and determine whether or not it is positive definite.

5. Find all values of the constant a such that the linear system $MX = B$ always has a solution for any $B \in \mathbb{R}^3$ if $M = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 1 & 1 \\ 1 & 1 & a \end{pmatrix}$.