FINAL EXAM

Math 136

name

Show all work; doing this may help you get more partial credit for problems done incorrectly. Use the backs of the test pages as necessary.

1. Define a function $f : \mathbb{R}^2 \to \mathbb{R}$ by $f(x, y) = \frac{x^5+y^7}{x^2+y^2}$ if $(x, y) \neq (0, 0)$ while $f(0, 0) = 0$. Determine with proof whether or not f is differentiable at $(0, 0)$. Ignore all points other than $(0, 0)$.

2. Find a 2×2 matrix with eigenvalues $1, 2$ and corresponding eigenvectors $\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \end{pmatrix}$, respectively.

3. Evaluate $\int_0^1 \int_{\sqrt{x}}^1 \sin y^3 \, dy \, dx$ by first changing the order of integration.

4. Every entry of a 3×3 matrix A is either 0 or 1. Determine the largest possible value of $\det A$, giving an explicit example of a matrix A having this largest determinant.

5. Set up an integral in spherical coordinates representing the mass of an object occupying the upper unit hemisphere whose density at any point equals the square of its distance from the xy-plane. Do NOT evaluate the integral.

6. Find the eigenvalues of the symmetric matrix $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix}$. (Two of the eigenvalues are obvious; work out eigenvectors with these eigenvalues and then use the existence of a third eigenvector with a different eigenvalue orthogonal to the first two eigenvectors.)

7. Let R be a region in $\mathbb{R}^3$ with volume V. Replacing every point $(x, y, z) \in R$ by the point $(x + 2y + 3z, 2x + 3y + 4z, x + y)$, we obtain a new region R' whose volume V' turns out to depend only on V and not on the shape of R. Work out a formula for V' in terms of V.

8. Find the singular values of the matrix $\begin{pmatrix} 1 & 2 \\ 1 & 0 \end{pmatrix}$.