MIDTERM #2

Math 135A

name

You must show all work for full credit. Use the backs of the test pages as necessary.

1. Given that $y_1(t) = t$ and $y_2(t) = te^t$ both satisfy the equation $t^2y'' - t(t+2)y' + (t+2)y = 0$, find the general solution to the inhomogeneous equation $t^2y'' - t(t+2)y' + (t+2)y = 2t^3$.

2. Find the equation of the unique plane passing through the points $(1, 1, 1), (2, 1, 2)$, and $(3, 2, 0)$.

3. Given the equation $\phi(t) + \int_0^t k(t-\xi)\phi(\xi)\, d\xi = f(t)$, where f and k and known functions and ϕ is an unknown function, take Laplace transforms of both sides to solve for $\Phi(s)$, the Laplace transform of $\phi(t)$, in terms of the Laplace transforms of f and k.

4. Find the equation of the osculating plane of the curve $(e^t, \cos t, 0)$ at any time t.

5. Use Laplace transforms to work out the solution to $y'' + y = u_\pi(t) - u_{3\pi}(t), y(0) = 0, y'(0) = 0$, where $u_\pi, u_{3\pi}$ are the unit step functions equal to 1 for $t \geq \pi, t \geq 3\pi$, respectively, while these functions are 0 at all other values of t.