FINAL EXAM

Math 135A

name

The questions are weighted equally. Use the backs of the test pages as necessary.

1. Let $f(x,y) = \frac{x^3+y^3}{x^2+y^2}$ if $(x,y) \neq (0,0)$ while $f(0,0) = 0$. Determine with proof whether or not f is differentiable at $(0,0)$. (First compute the partial derivatives of f at $(0,0)$ and then decide whether or not the definition of differentiability is satisfied with these values of the partial derivatives.)

2. Find a parametrization of the ellipse with equation $\frac{x^2}{4} + \frac{y^2}{9} = 1$ and integrate $x\,dy$ with respect to t over a suitable interval to find the area enclosed by this ellipse.

3. Find the set of values of x for which $\sum_{n=0}^{\infty}(x+4)^{-n}$ converges.

4. Given the linear differential equation $ay''+by'+cy=0$ with positive constant coefficients a, b, c, show that the 0 solution is asymptotically stable (i.e. all solutions $y(t)$ approach 0 as $t \to \infty$).

5. Use Laplace transforms to express the unique solution to $y^{(3)} - y = g(t)$ with $y(0) = y'(0) = y''(0) = 0$ as a convolution integral. Here $g(t)$ is a continuous function.

6. Let L_1, L_2 be the lines through $(2, 5, -3)$ and $(4, 9, 5)$, respectively, with respective direction vectors $(3, 7, 2)$ and $(2, 6, -12)$. Determine whether or not L_1 and L_2 intersect, and if so at which point.

7. Let $\sum_{n=0}^{\infty} a_n x^n$ be a nonzero power series solution to the differential equation $y' - x^2 y = 0$. Say as much as you can about the coefficients a_n.

8. Find the unit principal normal vector to the parametrized curve $(\cos t, \sin t, \cos 2t, \sin 2t)$.