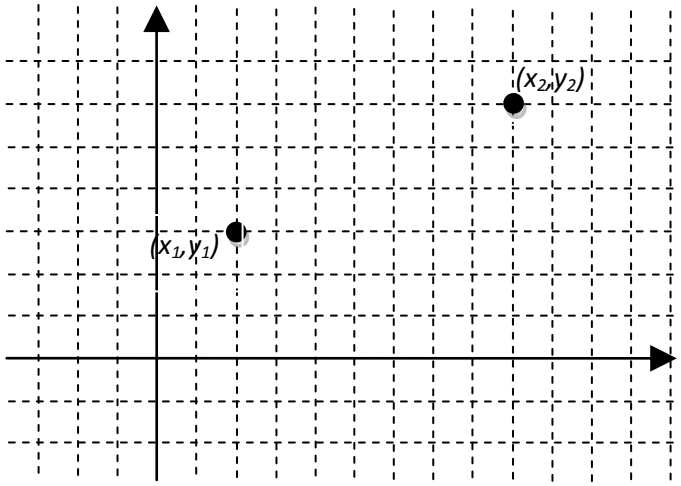
WMS Math Challenge

April 22, 2010

Driving Circles in Hybrid Taxicabs

Problem 1: In the following picture, two points (x_1,y_1) and (x_2,y_2) are plotted.



Part (a): Plot the point (x_2,y_1) in the grid.

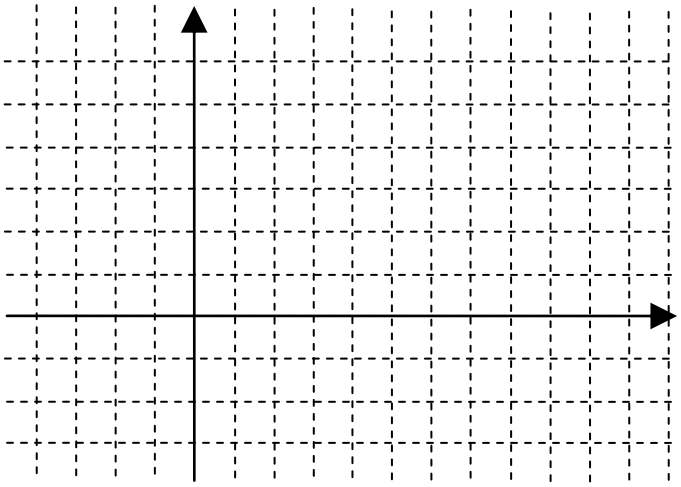
Part (b): Draw a straight line from (x_1,y_1) to (x_2,y_1). What is the length of this line?

Part (c): Draw a straight line from (x_2,y_1) to (x_2,y_2). What is the length of this line?

Part (d): Draw a straight line from (x_1,y_1) to (x_2,y_2). What is the length of this line?

Part (e): What is the distance $d(P,Q)$ between the points $P=(x_1,y_1)$ and $Q=(x_2,y_2)$.

Problem 2:



Part (a): Plot the point *(1,2)* in the grid shown above.

Part (b): Draw all the points that are 3 units away from the point *(1,2)*.

Part (c): What shape do you get?

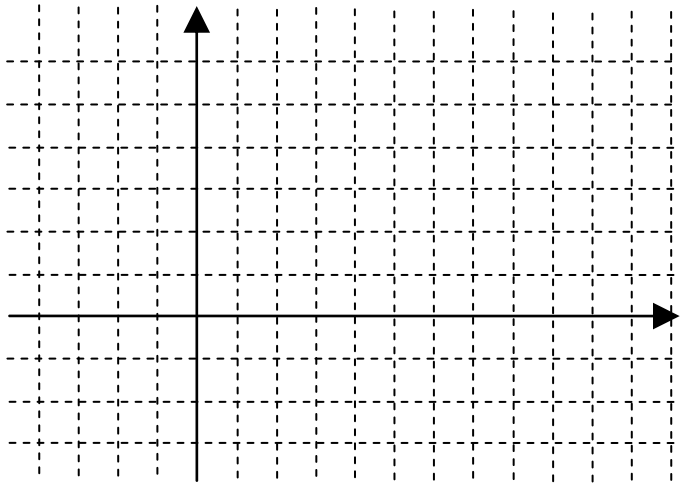
Problem 3: Let's define a new distance formula so that the distance d_1 from the point $P=(x_1,y_1)$ to the point $Q=(x_2,y_2)$ is

$$d_1(P,Q) = |x_2 - x_1| + |y_2 - y_1|.$$

Part (a): Compute the distance between the following points using our new distance formula:

1. *P=(3,4), Q=(2,1)* $d_1(P,Q) =$ _____
2. *P=(1,-1), Q=(3,5)* $d_1(P,Q) =$ _____
3. *P=(0,0), Q=(0,0)* $d_1(P,Q) =$ _____
4. *P=(-2,-2), Q=(8,10)* $d_1(P,Q) =$ _____

Part (b): Plot and label the points $P=(3,4)$ and $Q=(2,1)$ in the grid shown below.



Part (c): Plot and label the points $R=(3,2)$ and $S=(-1,3)$ in the above grid as well.

Part (d): Remember that we defined

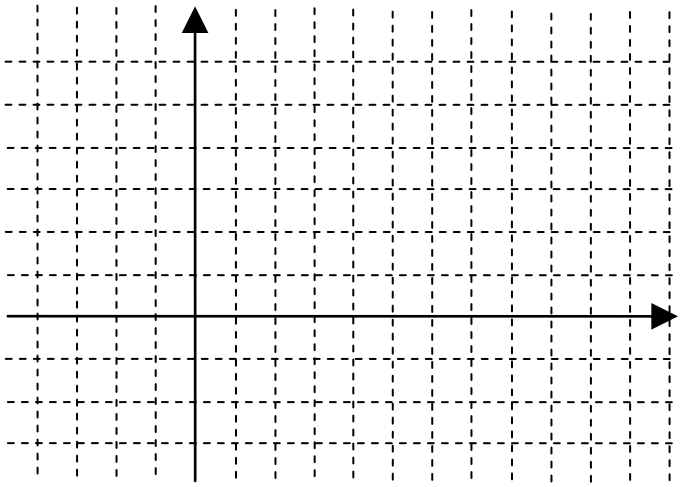
$$d_1(P,Q)=|3-2|+|4-1|.$$

What distance is represented by $|3-2|$? How about by $|4-1|$?

Part (e): Many people call the distance $d_1(P,Q)$ the *taxicab distance*. Why do you think that is?

Part (f): Compute $d_1(P,Q)$, $d_1(Q,S)$, and $d_1(P,S)$. What do you notice about $d_1(Q,S)+ d_1(S,P)$?

Problem 4:



Part (a): Plot the point *(1,2)* in the above grid.

Part (b): Plot all the points that lie 3 units away from the point *(1,2)* in the taxicab distance.

Part (c): What shape do you see now?